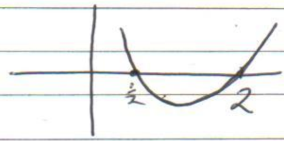
Ex 7.2

Q1 (iii)

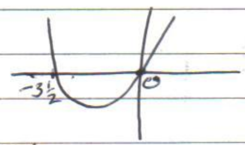
$$2x^2 - 5x + 2 < 0$$
$$(2x-1)(x-2) = 0$$
$$2x = 1 \qquad x = 2$$
Roots. $x = \frac{1}{2}$



Hence : $2x^2 - 5x + 2 < 0$
$$\frac{1}{2} < x < 2.$$

Q2 (iii)

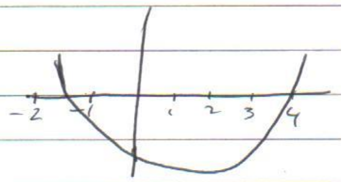
$$-2x^2 - 7x \geq 0$$
$$-2x^2 - 7x = 0$$
$$2x^2 + 7x = 0$$
$$x(2x+7) = 0$$
$$x = 0 \qquad 2x = -7$$
$$x = -7/2 = -3\tfrac{1}{2}$$



Hence : $-2x^2 - 7x \geq 0 \Rightarrow 2x^2 + 7x \leq 0.$
$$\Rightarrow -3\tfrac{1}{2} \leq x \leq 0$$

Q3 (ii)

$$2(x^2-6) \geq 5x$$
$$2x^2 - 12 - 5x = 0$$
$$2x^2 - 5x - 12 = 0$$
$$(2x+3)(x-4) = 0$$
$$2x = -3$$
Roots $x = -3/2 \qquad x = 4$



Hence $2(x^2-6) \geq 5x \Rightarrow x \leq -1\tfrac{1}{2}$ and $x \geq 4$

or $-1\tfrac{1}{2} \geq x \geq 4$

Q5 $x^2 - 6x + 2 \leq 0$

$x^2 - 6x + 2 = 0$ $a = 1,\ b = -6,\ c = 2$

$$x = \frac{6 \pm \sqrt{36 - 8}}{2}$$

$$x = \frac{6 \pm \sqrt{28}}{2}$$

$$x = \frac{6 \pm 2\sqrt{7}}{2}$$

$$x = 3 \pm \sqrt{7}$$

Hence $x^2 - 6x + 2 \leq 0$

$\Rightarrow \quad 3 - \sqrt{7} \leq x \leq 3 + \sqrt{7}$

Q6 $x^2 + (k+1)x + 1 = 0$ real roots

$\Rightarrow b^2 - 4ac \geq 0$

$(k+1)^2 - 4(1)(1) \geq 0$

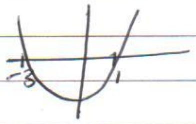
$k^2 + 2k + 1 - 4 \geq 0$

$k^2 + 2k - 3 \geq 0$

$k^2 + 2k - 3 = 0$

$(k + 3)(k - 1) = 0$

$k = -3 \quad k = 1$



Hence: $k^2 + 2k - 3 \geq 0$ $x \leq -3$ and $x > 1$

$\Rightarrow \quad -3 \geq x \geq 1$

Q7 $kx^2 + 4x + 3 + k = 0$ has real roots

$\Rightarrow b^2 - 4ac \geqslant 0$

$(4)^2 - 4(k)(3+k) \geqslant 0$

$16 - 12k - 4k^2 \geqslant 0$

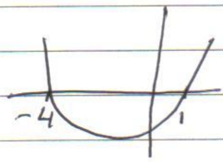
$4 - 3k - k^2 \geqslant 0$

$k^2 + 3k - 4 \leqslant 0.$

$k^2 + 3k - 4 = 0$

$(k + 4)(k - 1) = 0$

$k = -4 \qquad k = 1$



Hence $k^2 + 3k - 4 \leqslant 0 \Rightarrow -4 \leqslant x \leqslant 1$

Q8 $px^2 + (p+3)x + p = 0$ has real roots.

$\Rightarrow b^2 - 4ac \geqslant 0$

$(p+3)^2 - 4(p)(p) \geqslant 0$

$p^2 + 6p + 9 - 4p^2 \geqslant 0$

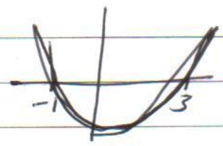
$-3p^2 + 6p + 9 \geqslant 0$

$p^2 - 2p - 3 \leqslant 0$

Solve $p^2 - 2p - 3 = 0$

$(p - 3)(p + 1) = 0$

$p = 3 \qquad p = -1$



Hence $p^2 - 2p - 3 \leqslant 0$

$\Rightarrow -1 \leqslant p \leqslant 3$

$x = -2$ is a root

$\Rightarrow p(-2)^2 + (p+3)(-2) + p = 0$

$4p - 2p - 6 + p = 0$

$3p = 6$

$p = 2.$

Q9 (i) $\frac{x+3}{x+2} < 2$

$\frac{x+3}{x+2} \times (x+2)^2 \leq 2(x+2)^2$

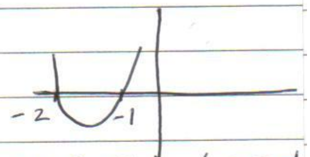
$(x+3)(x+2) \leq 2(x^2+4x+4)$

$x^2+3x+2x+6 \leq 2x^2+8x+8$

$0 \leq x^2+3x+2$

$(x+2)(x+1)$

$x=-2 \qquad x=-1$



Hence $x^2+3x+2 > 0 \Rightarrow x < -2$ and $x > -1$

$-2 > x > -1$

(ii) $\frac{x+5}{x-3} > 1$

$\frac{x+5}{x-3} \times (x-3)^2 > 1(x-3)^2$

$(x+5)(x-3) > 1(x^2-6x+9)$

$x^2-3x+5x-15 > x^2-6x+9$

$8x > 24$

$x > 3$

(iii) $\frac{2x-1}{x+3} > 3$

$\frac{2x-1}{x+3} \times (x+3)^2 > 3(x+3)^2$

$(2x-1)(x+3) > 3(x^2+6x+9)$

$2x^2+6x-x-3 > 3x^2+18x+27$

$-x^2-13x-30 > 0$

$x^2+13x+30 < 0$

$(x+3)(x+10) = 0 \qquad -10 < x < -3$

$x=-3 \qquad x=-10$

Q10 (ii) $\frac{1-2x}{4x+2} > 2$

$\frac{1-2x}{4x+2} \times (4x+2)^2 > 2(4x+2)^2$

$(1-2x)(4x+2) > 2(16x^2+16x+4)$

$4x+2-8x^2-4x > 32x^2+32x+8$

$-40x^2-32x-6 > 0$

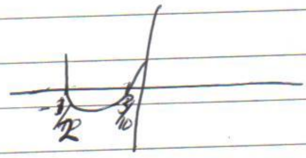
$20x^2+16x+3 < 0$

Solve $20x^2+16x+3 = 0$

$(10x+3)(2x+1) = 0$

$10x = -3 \qquad 2x = -1$

$x = -3/10 \qquad x = -1/2$



Hence: $20x^2+16x+3 < 0$

$\Rightarrow -1/2 < x < -3/10$

Q11 (ii) $\frac{2x-4}{x-1} < 1$

$\frac{2x-4}{x-1} \times (x-1)^2 < 1(x-1)^2$

$(2x-4)(x-1) < 1(x^2-2x+1)$

$2x^2-2x-4x+4 < x^2-2x+1$

$x^2-4x+3 < 0$

Solve $x^2-4x+3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \qquad x = 1$

Hence $x^2-4x+3 < 0$

$1 < x < 3$

Q12 (ii) $\frac{2x-3}{x-5} < \frac{3}{2}$

$\frac{2x-3}{x-5} \times (x-5)^2 < \frac{3}{2}(x-5)^2$

$(2x-3)(x-5) < \frac{3}{2}(x^2-10x+25)$

$2x^2-10x-3x+15 < \frac{3x^2-30x+75}{2}$

$4x^2-20x-6x+30 < 3x^2-30x+75$

$x^2+4x-45<0$

Solve $x^2+4x-45=0$

$(x-5)(x+9)=0$

$x=5 \quad x=-9$

Hence $x^2+4x-45<0$

$\Rightarrow \quad -9<x<5$

Q13 (i) from Graph $2x^2+4x > x^2-\cancel{x}-6$

at $x<-3$ and $x>-2$

$\Rightarrow \quad -3>x>-2$

(ii) $2x^2+4x > x^2-x-6$

$x^2+5x+6=0$

$(x+3)(x+2)=0$

$x=-3 \quad x=-2$

Hence $x^2+5x+6>0$

$-3>x>-2$

Q14 $x^2 + x + 1 > 0$ for all values of x

$\Rightarrow$ the x axis $\Rightarrow$ No Real roots

$\Rightarrow b^2 - 4ac < 0$

$a = 1 \quad b = 1 \quad c = 1$

$(1)^2 - 4(1)(1) < 0$

$1 - 4 < 0$

$-3 < 0$ True.

Hence $\cancel{x}^2 + x + 1 > 0$.

Q15 $f(t) = -11 + 13t - 2t^2$.

(i) $f(t) \leq 4 \Rightarrow -11 + 13t - 2t^2 \leq 4$

$-15 + 13t - 2t^2 \leq 0$

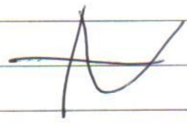
$2t^2 - 13t + 5 \geq 0$

Solve $2t^2 - 13t + 15 = 0$

$(2t - 3)(t - 5) = 0$

$2t = 3$

Roots $t = 3/2 \quad t = 5$



Hence $2t^2 - 13t + 5 \geq 0$

$3/2 \geq \cancel{t} \geq 5$.

(ii) $f(t) \geq 7 \quad -11 + 13t - 2t^2 \geq 7$

$2t^2 - 13t + 18 \leq 0$

Solve $2t^2 - 13t + 18 = 0$

$(2t - 9)(t - 2) = 0$

$t = 9/2 \quad t = 2$

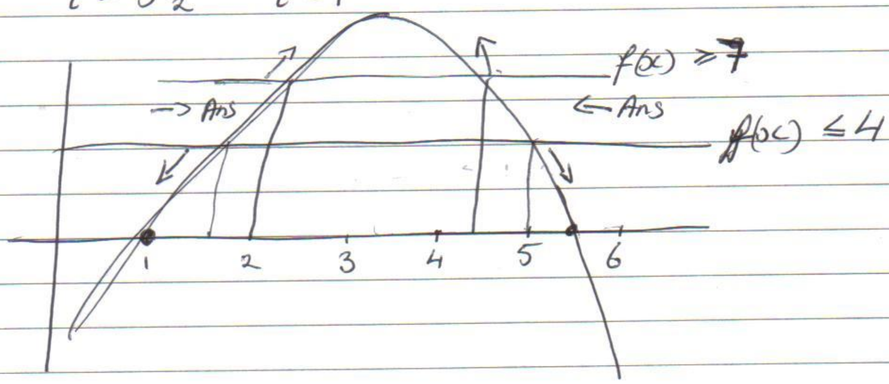
Hence $2t^2 - 13t + 18 \leq 0 \Rightarrow 2 \leq \cancel{t} \leq 4\frac{1}{2}$

Range that Satisfies $4 < f(x) < 7$.

$2t^2 - 13t + 11 = 0$

$(2t - 11)(t - 1) = 0$

$t = 5\tfrac{1}{2} \quad t = 1$



$\Rightarrow 1\tfrac{1}{2} < t < 2$ and $4\tfrac{1}{2} < t < 5$

Q16 (i) $x < -3$ and $x > -\tfrac{1}{2}$ $\Rightarrow$ $-3 > x > -\tfrac{1}{2}$

(ii) $x \leq 1$ and $x \geq 3$ $\Rightarrow$ $1 \geq x \geq 3$

(iii) $-1.5 \leq x \leq 0.5$

(iv) $-1 < x < 5$.

Q17 width $= (x-3)$ length $= x$

Ratio < 5

$$\Rightarrow \frac{x}{x-3} < 5$$

$$\frac{x}{x-3} \times (x-3)^2 < 5(x-3)^2$$

$$x(x-3) < 5(x^2-6x+9)$$

$$x^2-3x < 5x^2-30x+45$$

$$-4x^2+27x-45 < 0$$

$$4x^2-27x+45 > 0$$

Solve $(4x-15)(x-3) = 0$

$4x = 15$

$x = 15/4$ $\quad x = 3$

Hence $4x^2-27x+45 > 0$

$x < 3$ and $x > 3\frac{3}{4}$

$\Rightarrow$ $3 > x > 3\frac{3}{4}$

$x = 3$ not Valid as width $= x-3 = 3-3 = 0$

$\Rightarrow$ $x > 3\frac{3}{4}$

$\Rightarrow$ length $> 3\frac{3}{4}$

Width $> (x-3) = (3\frac{3}{4}-3) = 3/4$

Width $> 3/4$

Q18

All graphs are above x axis $\Rightarrow$ No real Roots

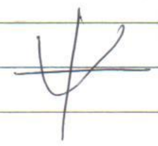
$\Rightarrow \quad b^2 - 4ac < 0$

$x^2 - 2px + p + 6 \qquad a = 1 \quad b = -2p \quad c = p+6$

$\Rightarrow \quad (-2p)^2 - 4(1)(p+6) < 0$

$4p^2 - 4p - 24 < 0$

$p^2 - p - 6 < 0$



solve $\quad p^2 - p - 6 = 0$

$(p-3)(p+2) = 0$

$p = 3 \qquad p = -2$

Hence $\quad p^2 - p - 6 < 0 \quad \Rightarrow \quad -2 < p < 3$

Q19 (i) Perimeter < 50

$2(x+3) + 2(x+2) < 50$

$2x + 6 + 2x + 4 < 50$

$4x < 40$

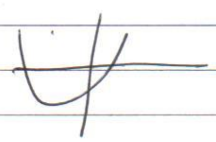
$x < 10$

(ii) Area > 12

$(x+3)(x+2) > 12$

$x^2 + 2x + 3x + 6 > 12$

$x^2 + 5x - 6 > 0$



Solve $\quad x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6 \qquad x = 1$

Hence $\quad x^2 + 5x - 6 > 0 \quad \Rightarrow \quad x < -6$ and $x > 1$

$\Rightarrow \quad -6 > x > 1$

-6 is not Valid $\quad \Rightarrow$ Ans $\quad x > 1$.

Q19 (iii) Perimeter $< 50 \Rightarrow x < 10$

Area $> 12 \Rightarrow x > 1$

$\Rightarrow$ for both $\Rightarrow 1 < x < 10$

Q20 $8 <$ Perimeter < 12

find 3^{rd} Side.

$$(hyp)^2 = x^2 + 3^2$$
$$hyp^2 = x^2 + 9$$
$$hyp = \sqrt{x^2+9}$$

$$8 < x + 3 + \sqrt{x^2+9} < 12$$

$8 < x + 3 + \sqrt{x^2+9}$

$5 - x < \sqrt{x^2+9}$ (sq both sides)

$25 - 10x + x^2 < x^2 + 9$

$16 < 10x$

$1{\cdot}6 < x$

$x + 3 + \sqrt{x^2+9} < 12$

$\sqrt{x^2+9} < 9 - x$

$x^2 + 9 < 81 - 18x + x^2$

$18x < 72$

$x < 4$

$$1{\cdot}6 < x < 4$$

But $x \in \mathbb{Z}$

$\Rightarrow$ Ans: $x = \{2, 3\}$.