

Exercise 7.3

Q1 (i) $|x+3|=1 \Rightarrow x+3=1$ or $x+3=-1$
 $x=-2$ $x=-4$

(ii) $|x-2|=4 \Rightarrow x-2=-4$ or $x-2=4$
 $x=-2$ $x=6$

(iii) $|2x-1|=5 \Rightarrow 2x-1=-5$ or $2x-1=5$
 $2x=-4$ $2x=6$
 $x=-2$ $x=3$

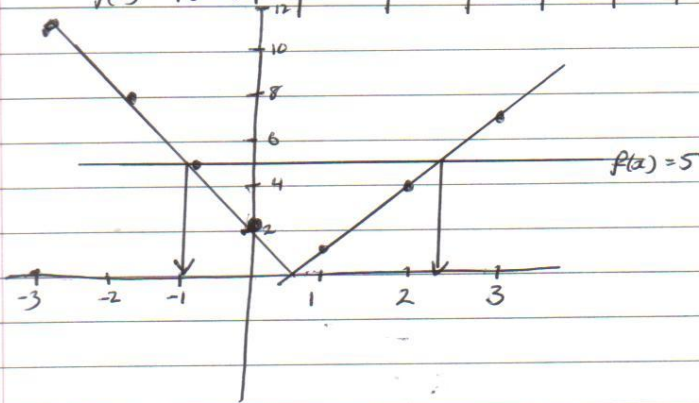
(iv) $|3x-2|=x \Rightarrow 3x-2=-x$ or $3x-2=x$
 $4x=2$ $2x=2$
 $x=1/2$ $x=1$

(v) $2|x-3|=2 \Rightarrow 2(x-3)=-2$ or $2(x-3)=2$
 $2x-6=-2$ $2x-6=2$
 $2x=4$ $2x=8$
 $x=2$ $x=4$

(vi) $|x-5|=|x+1|$ sq both sides.
 $x^2-10x+25 = x^2+2x+1$
 $-12x = -24$
 $x = 2$

Q2

x	-3	-2	-1	0	1	2	3
$f(x) = 3x-2 $	11	8	5	2	1	4	7



Solve $|3x-2|=5$

Sols: $x=-1, x=2\frac{1}{3}$

Q3

$f(x) = |x|$

$g(x) = |x-4|$

$h(x) = |x+3|$

$(4,0)$ $(5,1)$ $m=1$
 y intercept = -4
 $(-3,0)$ $(0,3)$ $m=1$

$f(-2) = |-2| = 2$ pt $(-2,2)$ is on $f(x)$

$h(-5) = |-5+3| = 2$ pt $(-5,2)$ is on $h(x)$

$g(2) = |2-4| = 2$ pt $(2,2)$ is on $g(x)$

Q4 ~~$f(x)$ y intercept = 1 pts $(0,1)$ $(1,2) \Rightarrow m = \frac{2-1}{1-0} = 1$~~
 ~~$f(x) = |x+1|$~~

~~$g(x)$ y intercept = 2 pts $(0,2)$ $(1,4) \Rightarrow m = \frac{4-2}{1-0} = 2$~~

~~$g(x) = |2x+2|$~~

$h(x)$ y intercept = 3 pts $(0,3)$ $(-1,0) \Rightarrow m = \frac{-3-0}{-1-0} = 3$

$h(x) = |3x+3|$

Q4 $f(x) = |ax + b|$

y intercept = 1 pts (0,1)(1,2) $\Rightarrow m = \frac{1}{1} = 1$

$f(x) = |x + 1|$

$g(x) = |ax + b|$ y intercept = 2 pts (0,2)(1,4) $\Rightarrow m = \frac{2}{1} = 2$

$g(x) = |2x + 2|$

$h(x) = |ax + b|$ y intercept = 3 pts (0,3)(-1,0) $\Rightarrow m = \frac{-3}{-1} = 3$

$h(x) = |3x + 3|$

Eqn: $x = -2$

$f(-2) = |-2 + 1| = 1$ pt (-2,1) is on $f(x)$

$g(-2) = |2(-2) + 2| = 2$ pt (-2,2) is on $g(x)$

$h(-2) = |3(-2) + 3| = 3$ pt (-2,3) is on $h(x)$

Q5 $f: x \rightarrow |x - 2|$

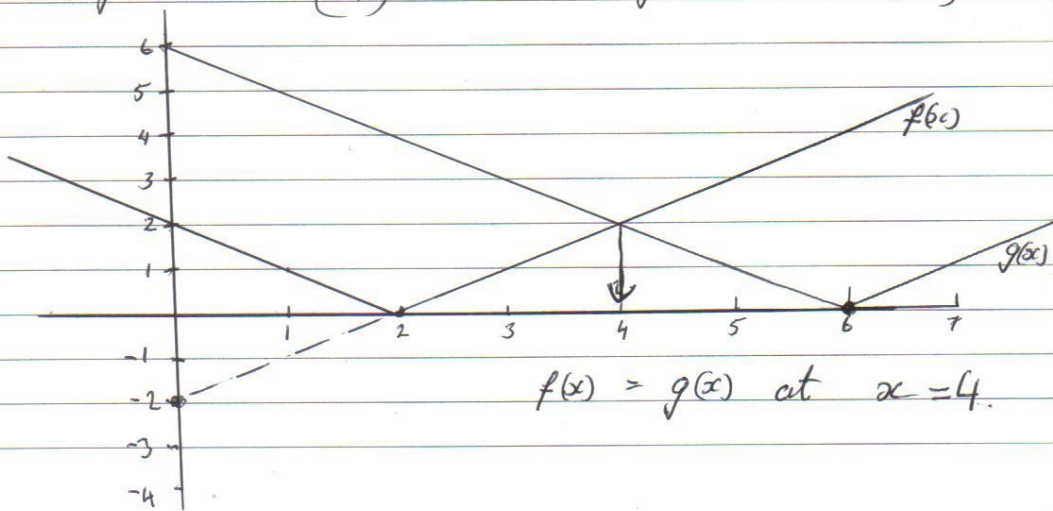
at $x = 0$ $y = -2$ (0, -2)

at $y = 0$ $x = 2$ (2, 0)

$g: x \rightarrow |x - 6|$

at $x = 0$ $y = -6$ (0, -6)

$y = 0$ $x = 6$ (6, 0)



Algebraically $|x-2| = |x-6|$ sq both sides.

$$x^2 - 4x + 4 = x^2 - 12x + 36$$

$$8x = 32$$

$$x = 4.$$

Q6 Rem: $|x| < 1 \Rightarrow -1 < x < 1$
 $|x| > 1 \Rightarrow x > 1$ or $x < -1$
 $\Rightarrow -1 > x > 1$

Q6 (i) $|x-6| < 2$
 $-2 < x-6 < 2$
 $-4 < x < 8$

(ii) $|x+2| \leq 4$
 $-4 \leq x+2 \leq 4$
 $-6 \leq x \leq 2$

(iii) $|2x-1| \geq 5$
 $-5 \geq 2x-1 \geq 5$
 $-4 \geq 2x \geq 6$
 $-2 \geq x \geq 3$

(iv) $|2x-1| \geq 11$
 $-11 \geq 2x-1 \geq 11$
 $-10 \geq 2x \geq 12$
 $-5 \geq x \geq 6$

(v) $|3x+5| < 4$
 $-4 < 3x+5 < 4$
 $-9 < 3x < -1$
 $-3 < x < -1/3$

Note the direction of the inequalities.

(vi) $|x-4| < 3$
 $-3 < x-4 < 3$
 $1 < x < 7$

Q7 (i) $|2x-1| \geq 7$
 $-7 \geq 2x-1 \geq 7$
 $-6 \geq 2x \geq 8$
 $-3 \geq x \geq 4$

(ii) $|3x+4| \leq |x+2|$ sq both sides.
 $9x^2 + 24x + 16 \leq x^2 + 4x + 4$
 $8x^2 + 20x + 12 \leq 0$
 $2x^2 + 5x + 3 \leq 0$
 $(2x+3)(x+1)$

Roots: $x = -3/2$ $x = -1$

\Rightarrow Sol: $-3/2 \leq x \leq -1$

(iii) $2|x-1| \leq |x+3|$ sq both sides
 $4(x^2 - 2x + 1) \leq x^2 + 6x + 9$
 $4x^2 - 8x + 4 \leq x^2 + 6x + 9$
 $3x^2 - 14x - 5 \leq 0$
 $(3x+1)(x-5)$

Roots: $x = -1/3$ $x = 5$

\Rightarrow Sol: $-1/3 \leq x \leq 5$

Q8

$$f(x) = |x| - 4$$

$$\text{at } x=0 \quad y=-4 \quad (0, -4)$$

$$\text{at } x=1 \quad y=-3 \quad (1, -3)$$

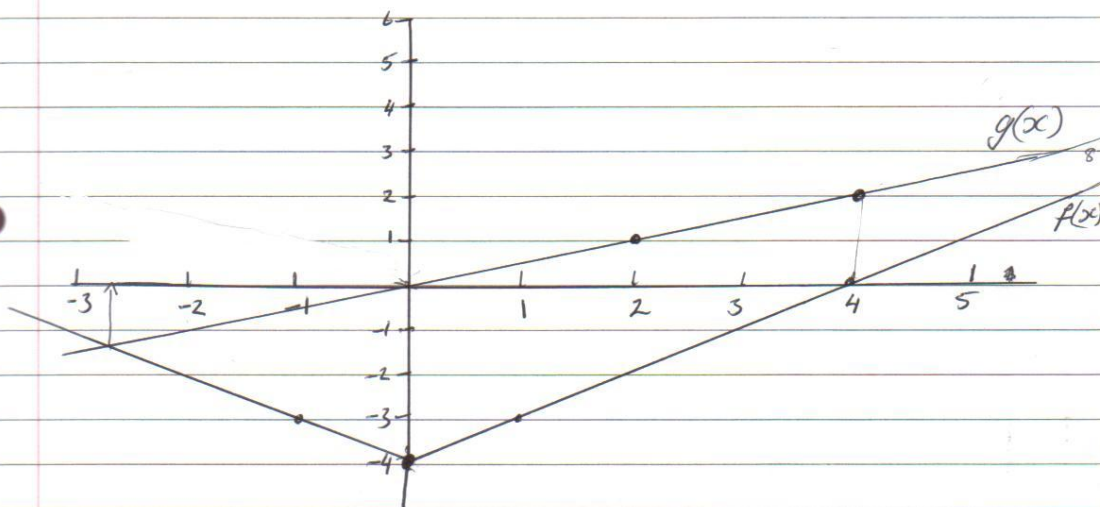
$$\text{at } x=-1 \quad y=-3 \quad (-1, -3)$$

$$\text{at } y=0 \quad x=4 \quad (4, 0)$$

$$g(x) = \frac{1}{2}x$$

$$\text{at } x=2 \quad y=1 \quad (2, 1)$$

$$\text{at } x=4 \quad y=2 \quad (4, 2)$$



Solve $|x| - 4 \leq \frac{1}{2}x$

algebraically:

$$|x| - 4 \leq \frac{1}{2}x$$

$$|x| \leq \frac{1}{2}x + 4$$

sq both sides: $x^2 \leq \frac{1}{4}x^2 + 4x + 16$

$$4x^2 \leq x^2 + 16x + 64$$

$$3x^2 - 16x - 64 = 0$$

$$(3x + 8)(x - 8)$$

roots $x = -8/3 \quad x = 8$

Sols: $-8/3 \leq x \leq 8$

or

$$|x| \leq \frac{1}{2}x + 4$$

$$-\frac{1}{2}x + 4 \leq x \leq \frac{1}{2}x + 4$$

$$-4 \leq x + \frac{1}{2}x \quad x \leq \frac{1}{2}x + 4$$

$$-4 \leq \frac{3}{2}x \quad \frac{1}{2}x \leq 4$$

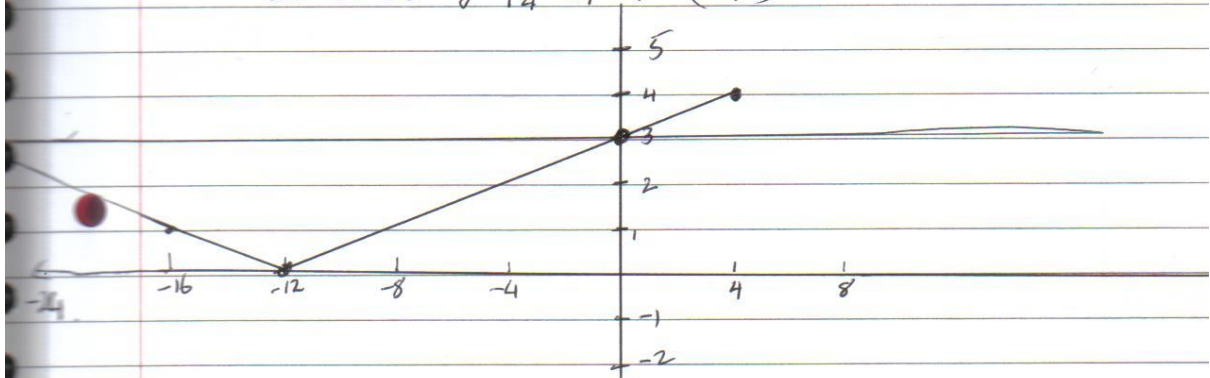
$$-4 \times \frac{2}{3} \leq x \quad x \leq 8$$

$$-8/3 \leq x$$

Sol: $-8/3 \leq x \leq 8$

Q9 $f(x) = \left| \frac{1}{4}x + 3 \right|$

at $x=0$ $y=3$ $(0,3)$
 at $x=4$ $y=4$ $(4,4)$
 at $y=0$ $\frac{1}{4}x = -3$
 $x = -12$ $(-12,0)$
 at $x=-16$ $y = \left| \frac{-16}{4} + 3 \right| = 1$ $(-16,1)$



Solve $\left| \frac{1}{4}x + 3 \right| \geq 3$ from graph. $x \geq 0$ and ~~$x \leq -24$~~
 $x \leq -24$

Algebraically: Sq both sides.

$$\frac{1}{16}x^2 + \frac{6}{4}x + 9 \geq 9$$

(x16) $x^2 + 24x \geq 0$

$$x(x+24)$$

$$x=0 \quad x=-24$$

$$-24 \geq x \geq 0$$

Q10 $|1+2x| < |x+2|$ sq both sides.

$$1+4x+4x^2 < x^2+4x+4$$
$$3x^2-3 < 0$$
$$\therefore 3(x^2-1) < 0$$
$$x^2-1 < 0$$
$$(x+1)(x-1)$$
$$x = -1 \quad x = 1$$

Sol: $-1 < x < 1$

Q11 $\left| \frac{1}{1+2x} \right| = 1$ sq both sides.

$$\frac{1}{1+4x+4x^2} = 1$$
$$1 = 1+4x+4x^2$$
$$0 = 4x^2+4x \quad (\div 4)$$
$$0 = x^2+x$$
$$x(x+1)$$
$$x = 0 \quad x = -1$$

Hence: Solve $\left| \frac{1}{1+2x} \right| < 1$

$$\frac{1}{1+4x+4x^2} < 1$$
$$1 < 1+4x+4x^2$$
$$0 < 4x+4x^2$$
$$-4x^2-4x < 0$$
$$4x^2+4x > 0$$
$$x^2+x > 0$$
$$x(x+1) > 0$$
$$x \neq 0 \quad x \neq -1$$

$$-1 > x > 0$$

Q12

- (i) $f(x) < h(x)$ $-4 < x < 2$
- (ii) $h(x) < f(x)$ $-4 > x > 2$
- (iii) $g(x) < f(x)$ $1.25 < x < 3.5$
- (iv) $g(x) < h(x) < f(x)$ $2 < x < 3$
- (v) $g(x) < f(x) < h(x)$ $1.25 < x < 2$
- (vi) $f(x) > h(x) > g(x)$ $2 < x < 3$
- (vii) $f(x) > g(x) > h(x)$ $3 < x < 3\frac{1}{2}$

Q13

(i) $\frac{x}{2x-1} < -2$

Mult both sides by $(2x-1)^2$
as do not know if is pos/neg

$$\frac{x}{2x-1} (2x-1)^2 < -2 (2x-1)^2$$

$$x(2x-1) < -2(4x^2 - 4x + 1)$$

$$2x^2 - x < -8x^2 + 8x - 2$$

$$10x^2 - 9x + 2 < 0$$

$$(5x - 2)(2x - 1) = 0$$

Roots : $x = \frac{2}{5}$ $x = \frac{1}{2}$

Sol $\frac{2}{5} < x < \frac{1}{2}$

Q13 (ii) $|x-3| = 2|x-1|$ sq both sides.

$$x^2 - 6x + 9 = 4(x^2 - 2x + 1)$$

$$x^2 - 6x + 9 = 4x^2 - 8x + 4$$

$$-3x^2 + 2x + 5 = 0$$

$$3x^2 - 2x - 5 = 0$$

$$(3x - 5)(x + 1) = 0$$

$$x = 5/3 \quad x = -1$$

(iii) $|x-1| < |2x+1| > 0$

$$|x-1| > |2x+1| \quad \text{sq both sides.}$$

$$x^2 - 2x + 1 > 4x^2 + 4x + 1$$

$$-3x^2 - 6x > 0$$

$$3x^2 + 6x < 0$$

$$(\div 3) \quad x^2 + 2x < 0$$

$$x(x+2) < 0$$

Roots $x = 0 \quad x = -2$

Sol: $-2 < x < 0$

[rem change inequality]