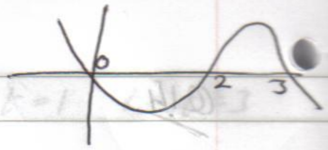
Revision Ex. (Core)



Q9 $x(2-x)(x-3) = (2x-x^2)(x-3)$

$$= 2x^2 - 6x - x^3 + 3x^2$$

$$= -x^3 + 5x^2 - 6x$$

$$\text{Area} = \int_0^2 (-x^3+5x^2-6x)\,dx + \int_2^3 (-x^3+5x^2-6x)\,dx$$

$$= \left[\frac{-x^4}{4} + \frac{5x^3}{3} - \frac{6x^2}{2}\right]_0^2 + \left[\frac{-x^4}{4} + \frac{5x^3}{3} - \frac{6x^2}{2}\right]_2^3$$

$$\left[\left(\frac{-(2)^4}{4} + \frac{5(2)^3}{3} - \frac{6(2)^2}{2}\right) - \cancel{\left(\frac{-0^4}{4} + \frac{5(0)^3}{3} - \frac{6(0)^2}{2}\right)}\right] + \left[\left(\frac{-(3)^4}{4} + \frac{5(3)^3}{3} - \frac{6(3)^2}{2}\right) - \left(\frac{-(2)^4}{4} + \frac{5(2)^3}{3} - \frac{6(2)^2}{2}\right)\right]$$

$$\left(-4 + \frac{40}{3} - 12\right) - (0) + \left(-\frac{81}{4} + 45 - 27\right) - \left(-4 + \frac{40}{3} - 12\right)$$

$$\left(-\frac{8}{3}\right) + \left(-\frac{9}{4} \overset{+}{-} -\frac{8}{3}\right)$$

$$\left(\frac{-8}{3}\right) + \left(-\frac{5}{12}\right)$$

Take absolute values

$$\frac{8}{3} + \frac{5}{12}$$

$$= \frac{37}{12} = 3\tfrac{1}{12} \text{ sq units}$$

Q1b $f(x) = x \sin 2x$

(product rule) $\frac{dy}{dx} = x . \cos 2x . 2 + \sin 2x (1)$

$= 2x \cos 2x + \sin 2x$

$\int 2x \cos 2x \, dx$

$\int (2x \cos 2x + \sin 2x) dx = x \sin 2x$

$\int (2x \cos 2x) dx + \int (\sin 2x) dx = x \sin 2x$

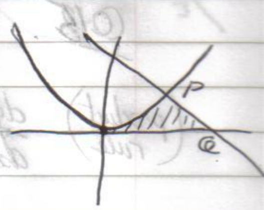
$\int 2x \cos 2x \, dx = x \sin 2x - \int (\sin 2x) dx$

$= x \sin 2x + \frac{\cos 2x}{2} + C$

Revision Ex (Advanced)

Q3 $\quad y = x^2 \qquad 2x + y = 15$

$\Rightarrow y = 15 - 2x$



(i) find P and Q.

Find P: $\quad x^2 = 15 - 2x$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5 \quad x = 3$

At P: $\quad x = 3 \qquad y = 15 - 2(3)$

$y = 9$

$P(3, 9)$

Find Q:

Q is where line cuts the x axis ie: $y = 0$

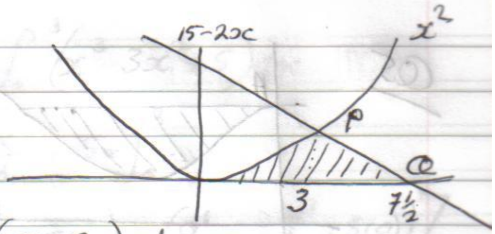
$0 = 15 - 2x$

$2x = 15$

$x = 7\frac{1}{2}$

$Q(7\frac{1}{2}, 0)$.

(ii) shaded area.



$$\text{Area} = \int_0^3 x^2 \, dx + \int_3^{7\frac{1}{2}} (15-2x) \, dx$$

$$= \left[\frac{x^3}{3}\right]_0^3 + \left[15x - \frac{2x^2}{2}\right]_3^{7\frac{1}{2}}$$

$$= \left[\left(\frac{(3)^3}{3}\right) - \left(\frac{0^3}{3}\right)\right] + \left[\left(15(7\tfrac{1}{2}) - (7\tfrac{1}{2})^2\right) - \left(15(3) - (3)^2\right)\right]$$

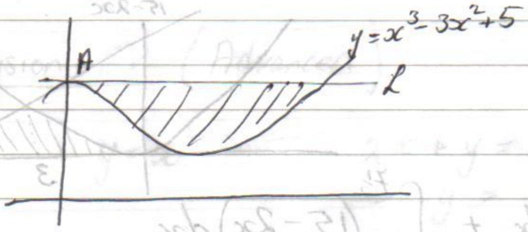
$$(9 - 0) + \left[(112.5 - 56.25) - (45 - 9)\right]$$

$$9 + (56.25 - 36)$$

$$9 + 20.25$$

$$= 29.25 \text{ sq units}$$

Q5



A is a max $\Rightarrow \frac{dy}{dx} = 0$

$y = x^3 - 3x^2 + 5$

$\frac{dy}{dx} = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

$y = x^3 - 3x^2 + 5.$

$x = 0 \Rightarrow y = 5$

$A(0, 5)$

find 2^{nd} pt: Where line $y = 5$ meets curve $y = x^3 - 3x^2 + 5.$

$5 = x^3 - 3x^2 + 5$

$0 = x^3 - 3x^2$

$0 = x^2(x-3)$

$x = 0 \quad x = 3.$

$$\text{Shaded Area} = \int_0^3 (5)\,dx - \int_0^3 (x^3 - 3x^2 + 5)\,dx$$

$$= \left[5x\right]_0^3 - \left[\frac{x^4}{4} - \frac{3x^3}{3} + 5x\right]_0^3$$

$$\left[5(3) - 5(0)\right] - \left[\left(\frac{3^4}{4} - \frac{3(3)^3}{3} + 5(3)\right) - \left(\frac{0^4}{4} - \frac{3(0)^3}{3} + 5(0)\right)\right]$$

$$15 - \left(\frac{33}{4}(1) - 0\right)$$

$$15 - 8.25 = 6.75 \text{ sq units.}$$

Q9

(i) $f(x) = x^2 \ln 3x$

$$f'(x) = x^2 \cdot \frac{3}{3x} + \ln 3x\,(2x)$$

$$= x + 2x \ln 3x.$$

(ii) $\int (2x \ln 3x + x)\,dx = x^2 \ln 3x$

$$\int (2x \ln 3x)\,dx + \int x\,dx = x^2 \ln 3x$$

$$\int (2x \ln 3x)\,dx = x^2 \ln 3x - \int x\,dx$$

$$\int (2x \ln 3x)\,dx = x^2 \ln 3x - \frac{x^2}{2} + C$$

Q11 $K = 5v^2$

$$\text{Average} = \frac{1}{7-1}\int_1^7 5v^2 \, dv$$

$$= \frac{1}{6}\left[\frac{5v^3}{3}\right]_1^7$$

$$= \frac{1}{6}\left[\frac{5(7)^3}{3} - \frac{5(1)^3}{3}\right]$$

$$= \frac{1}{6}\left[\frac{1715}{3} - \frac{5}{3}\right]$$

$$= \frac{1}{6}\left[\frac{1710}{3}\right] = 95 \text{ joules.}$$

$s \leftrightarrow v \leftrightarrow a$

Q12 $\quad a = 6t + 10 \quad$ rest $\Rightarrow v = 0$

$s = 3$

(i)
$$v = \int (6t + 10)\,dt$$
$$v = \frac{3\cancel{6}t^2}{\cancel{2}} + 10t + C = 3t^2 + 10t + C$$

$v = 0$ at $t = 0$ (at rest)
$$0 = 3(0)^2 + 10(0) + C \Rightarrow C = 0$$

$$v = 3t^2 + 10t$$

after 5 sec $\quad v = 3(5)^2 + 10(5)$
$$= 75 + 50$$
$$= 125 \text{ m/sec}$$

(ii) $\quad s = \int 3t^2 + 10t$
$$s = \frac{\cancel{3}t^3}{\cancel{3}} + \frac{{}^{5}\cancel{10}t^2}{\cancel{2}} + C$$
$$s = t^3 + 5t^2 + C$$

at $t = 0 \quad s = 3$
$$3 = (0)^3 + 5(0)^2 + C \Rightarrow C = 3$$
$$s = t^3 + 5t^2 + 3$$

(iii) after 3 sec! $\quad s = (3)^3 + 5(3)^2 + 3$
$$= 27 + 45 + 3 = 75 \text{ m}$$

(iv) Average Speed $= \frac{1}{4-1}\int_1^4 (3t^2 + 10t)\,dt$

$= \frac{1}{3}\left[\frac{\cancel{3}t^3}{\cancel{3}} + \frac{{}^{5}\cancel{10}t^2}{\cancel{2}}\right]_1^4 = \frac{1}{3}\left[\left((4)^3 + 5(4)^2\right) - \left((1)^3 + 5(1)^2\right)\right]$

$= \frac{1}{3}\left[144 - 6\right] = 46 \text{ m/sec}.$

Rev Ex Extended Response

Q2

(i) $y = 2x + \frac{8}{x^2} - 5$

$\Rightarrow y = 2x + 8x^{-2} - 5$

find pt P: $x = 1 \quad y = 2(1) + 8(1)^{-2} - 5 = 5 \quad P(1, 5)$

find pt Q $\quad x = 4 \quad y = 2(4) + \frac{8}{4^2} - 5 = \frac{7}{2} \quad Q(4, \frac{7}{2})$

Eqn of line PQ: Slope $= \frac{\frac{7}{2} - 5}{4 - 1} = -\frac{1}{2}$

$(1,5), m = -\frac{1}{2}$: $y - 5 = -\frac{1}{2}(x - 1)$

$$2y - 10 = -x + 1$$
$$2y = -x + 11$$
$$y = \frac{-x + 11}{2}$$

Shaded Area = Area under line − Area under Curve

$$= \int_1^4 \frac{1}{2}(-x + 11)\, dx - \int_1^4 (2x + 8x^{-2} - 5)\, dx$$

$$= \frac{1}{2}\left[\frac{-x^2}{2} + 11x\right]_1^4 - \left[\frac{2x^2}{2} + \frac{8x^{-1}}{-1} - 5x\right]_1^4$$

$$= \frac{1}{2}\left[-\frac{x^2}{2} + 11x\right]_1^4 - \left[x^2 - 8x^{-1} - 5x\right]_1^4$$

$$= \frac{1}{2}\left[\left(-\frac{4^2}{2} + 11(4)\right) - \left(-\frac{(1)^2}{2} + 11(1)\right)\right] - \left[\left((4)^2 - 8(4)^{-1} - 5(4)\right) - \left(1^2 - 8(1)^{-1} - 5(1)\right)\right]$$

$$\frac{1}{2}\left(36 - \frac{21}{2}\right) - \left[-6 - (-12)\right]$$

$$= \frac{1}{2}\left(\frac{51}{2}\right) - [6] = \frac{51}{4} - 6 = \frac{27}{4} \text{ sq. units.}$$

(b) $y = 2x + \frac{8}{x^2} - 5$ Increasing $\Rightarrow \frac{dy}{dx} >$

$y = 2x + 8x^{-2} - 5$

$\frac{dy}{dx} = 2 - 16x^{-3}$

$= 2 - \frac{16}{x^3}$

When $x > 2$ $\quad 2 - \frac{16}{x^3} > 0 \Rightarrow$ Increa

slope is posi

(c) turning pt $\Rightarrow \frac{dy}{dx} = 0$

$2 - \frac{16}{x^3} = 0$

$2x^3 - 16 = 0$

$x^3 - 8 = 0$

$x^3 = 8$

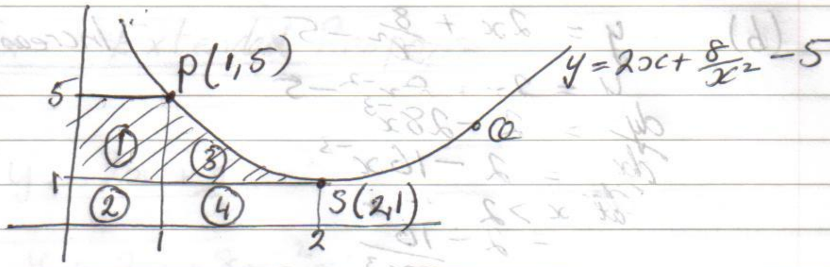
$x = 2.$

find y

$y = 2(2) + \frac{8}{(2)^2} - 5 = 1$

$\Rightarrow S(2, 1)$

(d)



Shaded Area = Rec (1) + (3)

$4 \times 1 = \underline{4}$

$\left[\text{Under curve}\right]_1^2 - (4)$

$1 \times 1 = \underline{1}$

find (3) $\int_1^2 \left(2x + 8x^{-2} - 5\right) dx - 1$

$= \left[\frac{2x^2}{2} + \frac{8x^{-1}}{-1} - 5x\right]_1^2 - 1$

$= \left(x^2 - \frac{8}{x} - 5x\right]_1^2 - 1$

$= \left[\left((2)^2 - \frac{8}{2} - 5(2)\right) - \left((1)^2 - \frac{8}{1} - 5(1)\right)\right] - 1$

$= \left[(4 - 4 - 10) - (1 - 8 - 5)\right] - 1$

$\left[(-10) - (-12)\right] - 1$

$(-10 + 12) - 1$

$2 - 1 = 1$ Area (3).

$\Rightarrow$ Shaded Area $= 4 + 1 = 5$ sq units.

Q4 $\frac{dV}{dt} = 120 + 26t - t^2$

(a) Initial $\Rightarrow t=0$ at $t=0$ $\frac{dv}{dt} = 120$.

Twice initial rate $= 240$

$240 = 120 + 26t - t^2$

$120 = 26t - t^2$

$t^2 - 26t + 120 = 0$

$(t-6)(t-20) = 0$

$t=6 \quad t=20$

at $t = 6$ sec or $t = 20$ sec

(b) $V = \int (120 + 26t - t^2)\, dt$

$= 120t + \frac{26t^2}{2} - \frac{t^3}{3} + C$

$= 120t + 13t^2 - \frac{t^3}{3} + C$

$V=0$ when $t=0$ $\Rightarrow 0 = 120(0) + 13(0)^2 - \frac{(0)^3}{3} + C$

$\Rightarrow C = 0$

$\therefore V = 120t + 13t^2 - \frac{t^3}{3}$

(c)

Initially $V = 1500$ lit $\quad t = 30$ mins

$V = 120(30) + 13(30)^2 - \frac{(30)^3}{3} = 6300$ litre

Total water in tank after 30min $= 1500 + 6300 = 78$

Tank holds 7000 lit $\Rightarrow$ litres lost $= 7800 - 7000$

$= 800$ litres.