Revision Exercise (Core)

Q1 (i) $\frac{12m^2n^3}{(6m^4n^5)^2} = \frac{12m^2n^3}{36m^8n^{10}} = \frac{1}{3m^6n^7}$

(ii) $\frac{3+\frac{1}{x}}{\frac{5}{x}+4}$ $(\times x)$ $\frac{3x+1}{5+4x}$

(iii) $\frac{2+\frac{x}{2}}{x^2-16}$ $\frac{\frac{4+x}{2}}{(x+4)(x-4)} = \frac{(4+x)}{2} \times \frac{1}{(x+4)(x-4)} = \frac{1}{2(x-4)}$

Q2 (i) $y = x+4$
$5y+2x=6$

$-x+y=4$ $(\times 2)$
$2x+5y=6$
$-2x+2y=8$
$2x+5y=6$
$7y=14$
$\boxed{y=2}$

$y=x+4$
$2=x+4$
$\boxed{-2=x}$ $(-2, 2)$

(ii) $3x+y=7$ $y=(7-3x)$
$x^2+y^2=13$
$x^2+(7-3x)^2=13$
$x^2+49-42x+9x^2=13$
$10x^2-42x+36=0$ $(\div 2)$
$5x^2-21x+18=0$
$(5x-6)(x-3)=0$
$5x=6$ $x=3$
$x=6/5$

$y=7-3x$
$y=7-3(\frac{6}{5})$
$y=7-\frac{18}{5}$
$y=7-3\frac{3}{5}=3\frac{2}{5}$
$(\frac{6}{5}, 3\frac{2}{5})$

$y=7-3(3)$
$y=7-9$
$y=-2$
$(3,-2)$

3.

$$x-3 \overline{)\, x^3 - x^2 - 7x + 3}$$

$x^2 + 2x - 1$

$-x^3 \pm 3x^2$

$2x^2 - 7x$

$-2x^2 \pm 6x$

$-x + 3$

$\pm x \mp 3$

0

4

$$x-2 \overline{)\, 3x^4 - 0x^3 - 9x^2 + 27x - 66}$$

$3x^3 + 6x^2 + 3x + 33$

$-3x^4 \pm 6x^3$

$6x^3 - 9x^2$

$-6x^3 \pm 12x^2$

$3x^2 + 27x$

$-3x^2 \pm 6x$

$33x - 66$

$-33x \pm 66$

0

5

(i) $x^4 - 9x^2 = 0$

$(x^2 - 3x)(x^2 + 3x) = 0$

$x^2 - 3x = 0$ $\quad$ $x^2 + 3x = 0$

$x(x-3) = 0$ $\quad$ $x(x+3) = 0$

$x = 0 \quad x = 3$ $\quad$ $x = 0 \quad x = -3$

OR

$x^2(x^2 - 9) = 0$

$x^2(x+3)(x-3) = 0$

$x = 0 \quad x = -3 \quad x = 3$

(ii) $(2x-1)^3(2-x) = 0$

$(2x-1)^3 = 0$ $\quad$ $(2-x) = 0$

$(2x-1)(4x^2 + 2x + 1)$ $\quad$ $2 = x$

$2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

6 $4x^2 + 20x + k$

$(2x + \sqrt{k})^2 = 4x^2 + 4x\sqrt{k} + k$

$\Rightarrow 4\sqrt{k} = 20$

$\sqrt{k} = 5$

$k = 25.$

$4x^2 + 20x + 25$

~~$(2x+5)$~~ $(2x+5)^2$

$4x^2 + 20x + 25$ ✓

7 (i) $(3 - \sqrt{2})^2 = a - b\sqrt{2}$

$9 - 6\sqrt{2} + 2 = a - 5\sqrt{2}$

$11 = a \qquad -6\sqrt{2} = -b\sqrt{2}$

$6 = b$

(ii) $\frac{1 - \sqrt{2}}{1 + \sqrt{2}} = a\sqrt{2} - b$

$\frac{1-\sqrt{2}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = a\sqrt{2} - b$

$\frac{1 - \sqrt{2} - \sqrt{2} + 2}{1 - \sqrt{2} + \sqrt{2} - 2} = a\sqrt{2} - b$

$\frac{3 - 2\sqrt{2}}{-1} = a\sqrt{2} - b$

$-3 + 2\sqrt{2} = a\sqrt{2} - b$

$-3 = -b \qquad 2\sqrt{2} = a\sqrt{2}$

$3 = b \qquad 2 = a$

Q8 $x^3 - 27 = (x-3)(x^2+3x+9)$

Q9 $p(x-q)^2 + r = 2x^2 - 12x + 5$

$p(x^2 - 2xq + q^2) + r = 2x^2 - 12x + 5$

$px^2 - 2pqx + pq^2 + r = 2x^2 - 12x + 5$

$p = 2$ $\quad -2pq = -12$ $\quad pq^2 + r = 5$

$2(2)q = 12$ $\quad (2)(3)^2 + r = 5$

$q = 3$ $\quad 18 + r = 5$

$r = -13$

10 A: $3x + 5y - z = -3$

B: $2x + y - 3z = -9$

C: $x + 3y + 2z = 7$

3A: $9x + 15y - 3z = -9$

B: $-2x - y - 3z = -9$

$7x + 14y = 0$

2A: $6x + 10y - 2z = -6$

C: $x + 3y + 2z = 7$

$7x + 13y = 1$

$7x + 14y = 0$

$-7x - 13y = -1$

$y = -1$

$7x + 14y = 0$

$7x + 14(-1) = 0$

$7x = 14$

$x = 2$

$3x + 5y - z = -3$

$3(2) + 5(-1) - z = -3$

$6 - 5 - z = -3$

$-z = -4$

$z = 4$

$(2, -1, 4)$

11. $(b+1)^3 - (b-1)^3$

$(b+1)(b^2+2b+1) - (b-1)(b^2-2b+1)$

$(b^3+2b^2+b+b^2+2b+1) - (b^3-2b^2+b-b^2+2b-1)$

$b^3+3b^2+3b+1 - b^3+2b^2-b+b^2-2b+1$

$6b^2+2$

12. (i) 3, 12, 27, 48, 75

1st Diff 9, 15, 21, 27

2nd Diff 6 6 6 ⇒ Quadratic.

$3x^2+3$ Test.: 3, 6, 15, 30 Incorrect.
(+6, +12, +18)

$3x^2+2x+3$ Test.: 3, 12, 27 -- Correct.

(ii) 5, 20, 45, 80, 125.

1st Diff 15, 25, 35, 45

2nd Diff 10 10 10 ⇒ Quadratic

$5x^2+5$ Test 5, 10, 25, 50 Incorrect.
(5, 15, 25)

	5	20	45	80	125
$-5x^2$	0	5	20	45	80
	5	15	25	35	45
1st Diff	10	10	10	10	⇒ Linear.

$10x+5$.

Ans $5x^2+10x+5$ Test.: 5, 20, 45, 80 correct.

~~45~~ 45 30 5

(iii) 0.5, 2, 4.5, 8, 12.5 ...

1st Diff 1.5 2.5 3.5 4.5

2nd Diff 1 1 1 ⇒ Quadratic

$\frac{1}{2}x^2 + 0.5$ Test. 0.5, 1, 2.5, 5 Incorrect.

0.5 1.5 2.5 3x

0.5, 2, 4.5, 8, 12.5

$-\frac{1}{2}x^2$ 0 0.5 2 4.5 8

0.5 1.5 2.5 3.5 4.5

1 # # # ⇒ linear.

$x + 0.5$

Ans: $\frac{1}{2}x^2 + x + 0.5$

13 6, 12, 20, 30, 42 n^2+3n+2

1st Diff 6 8 10 12

2nd Diff 2 2 2 ⇒ Quadratic

$x^2 + 6$ Test: 6 7 10 15 incorrect

6 12 20 30 42

$-x^2$ 0 -1 -4 -9 -16

(6) 11 16 21 26

(5) 5 5 5 ⇒ linear

$5x + 6$

⇒ polynomial is $x^2 + 5x + 6$

check 6, 12, 20, 30 ... correct

Term 100 = $f(99) = (99)^2 + 5(99) + 6 = 10,302$

Q14

$3w - 3 = 2L$

$4L - 12 = 2w + 2L$

$3w - 2L = 3$

$-2w + 2L = 12$

$w = 15$

$3w - 3 = 2L$

$3(15) - 3 = 2L$

$42 = 2L$

$21 = L$

Q15

$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$ $\qquad m = \frac{v-r}{r-u}$

$(\times uvr)$ $\quad vr + ur = 2uv$

$r(v+u) = 2uv$

$r = \frac{2uv}{v+u}$

$m = \frac{v - \frac{2uv}{v+u}}{\frac{2uv}{v+u} - u}$ $\qquad (\times\ v+u)$

$m = \frac{v(v+u) - 2uv}{2uv - u(v+u)} = \frac{v^2 + vu - 2uv}{2uv - uv - u^2}$

$= \frac{v^2 - uv}{uv - u^2} = \frac{v(v-u)}{u(v-u)}$

$= \frac{v}{u}$.

Rev Ex Advanced.

Q1 ~~4~~, ~~10~~, 1, 3, 6, 10

1st Diff 2 3 4

2nd Diff 1 1 ⇒ Quadratic

$\frac{1}{2}x^2 + 1$. check 1, 1½, 3, 5½ incorrect.

+½ +3 4½

1st 2nd 3rd

$\boxed{\frac{1}{2}x^2 + 1\frac{1}{2}x + 1}$ check

$f(0) = 1$

$f(1) = \frac{1}{2} + 1\frac{1}{2} + 1 = 3$

$f(2) = 2 + 3 + 1 = 6$

$f(3) = 4.5 + 4.5 + 1 = 10$ correct.

Q2 $55x + 25(1) = 35(x+1)$

$55x + 25 = 35x + 35$

$20x = 10$

$x = \frac{1}{2}$ m³.

Q3

$x + y = 8.4$

$60x + 40y = 50(8.4)$

$x + y = 8.4$

$60x + 40y = 420$ (÷ 20)

$x + y = 8.4$ (×2)

$3x + 2y = 21$

$-2x - 2y = -16.8$

$3x + 2y = 21$

$x = 4.2$

$4.2 + y = 8.4$

$\Rightarrow y = 4.2$.

Q4 $(3p-2t)x + r - 4t^2 = 0$ (r in terms of p)

equate coefficients.

$3p - 2t = 0$

$3p = 2t$

$\frac{3p}{2} = t$

$r - 4t^2 = 0$

$r - 4\left(\frac{3p}{2}\right)^2 = 0$

$r - 4\left(\frac{9p^2}{4}\right) = 0$

$r - 9p^2 = 0$

$r = 9p^2$ QED.

Q5 $\frac{x + y^2}{x^2} + \frac{x-1}{x} = -1$ $(\times x^2)$

$x + y^2 + x(x-1) = -1(x^2)$

$x + y^2 + x^2 - x = -x^2$

$2x^2 + y^2 = 0$

Ratio $x^2 : y^2$

$2x^2 = -y^2$

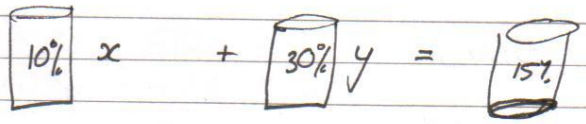
$2 : -1$

$\Rightarrow -\frac{1}{2}$

Q6

need 15% Sol.

a 10% sol + 30% sol = 10 Lit.



$10x + 30y = 150$

$\boxed{x + 3y = 15}$

Total $\boxed{x + y = 10}$

$$\begin{array}{r} x + 3y = 15 \\ -\ x + y = 10 \\ \hline 2y = 5 \end{array}$$

$y = 5/2 = 2.5$ of 30% Sol.

$x + y = 10$

$x + 2.5 = 10$

$x = 7.5$ of 10% Sol.

Q7

$S = \frac{D}{T}$ $T = \frac{D}{S}$

Brian a sec for 1m $s = \frac{1}{a}$ $\Rightarrow$ 50a to run 50m

Luke b sec for 1m $s = \frac{1}{b}$

Time = $\frac{\text{Dis}}{\text{Speed}}$ Brian = $\frac{50}{a}$ Luke = $\frac{50}{b}$

Luke wins by 1 sec.

$\frac{50}{b} - \frac{50}{a} = 1$ (x ab)

$\frac{50}{b} - \frac{47}{a} = 0.1$ (x ab)

Day 2
Luke = $\frac{50}{b}$
Brian = $\frac{47}{a}$

$50a - 50b = ab$

$-50a + 47b = -0.1ab$

$-3b = 0.9ab$ (÷b)

$-3 = 0.9a$

$-3/0.9 = a$

$-\frac{10}{3} = a$

Rev Ex Extended-Response

Q1 Adults €5 children <6 €2.50

Last yr $x + y = 13,000$ one price

~~$x + y = 548$~~
~~$5x +$~~

$$x + y = 13000$$
$$5x + 2.5y = 2460 \quad (\div 2.5)$$
$$x + y = 548$$
$$2x + y = 984$$
$$-x + y = 548$$

(i) $x = 436$ adults.

$$436 + y = 548$$

(ii) $y = 112$ children.

(iii) $\frac{436}{548} = 0.7956$ of Total is adults.

$x + y = 13000$

$\left(\frac{436}{548} \times 13000\right)$ adults @ €5 each $\Rightarrow$ €5171 5.33

$\left(\frac{112}{548} \times 13000\right)$ children @ €2.50 each $\Rightarrow$ €6642.34

Total $\Rightarrow$ 58357.67

$\Rightarrow$ 58358.

Q2

x: standard 2hrs Manu + 1hr finish

y: Delux 2.5hrs Manu + 1.5hr finish

48 hrs MANU & 26 hrs finishing

(i) $2x + 2.5y = 48$ all Manu info into one eqn

(ii) $x + 1.5y = 26$ (x2)

$-2x + -2.5y = -48$

$2x + 3y = 52$

$.5y = 4$

$y = 4/.5 =$ **8 deluxe**

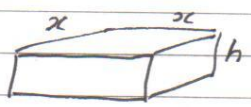
$x + 1.5y = 26$

$x + 1.5(8) = 26$

$x + 12 = 26$

$x =$ **14 standard**

Q3



Vol = 40.

$V = L \times W \times h$

$40 = x \times x \times h$

$40 = x^2 h$

$\frac{40}{x^2} = h$

$SA = 2(x^2) + 2(hx) + 2(hx)$

$= 2x^2 + 4hx.$

$= 2x^2 + 4\left(\frac{40}{x^2}\right)x$

$= 2x^2 + \frac{160}{x}$

(iii) $S = 2x^2 + \frac{160}{x}$

$x = 1$ $\quad S = 2(1)^2 + \frac{160}{1} \Rightarrow S = 162 \quad (1, 162)$

$x = 2$ $\quad S = 2(2)^2 + \frac{160}{2} \Rightarrow S = 88 \quad (2, 88)$

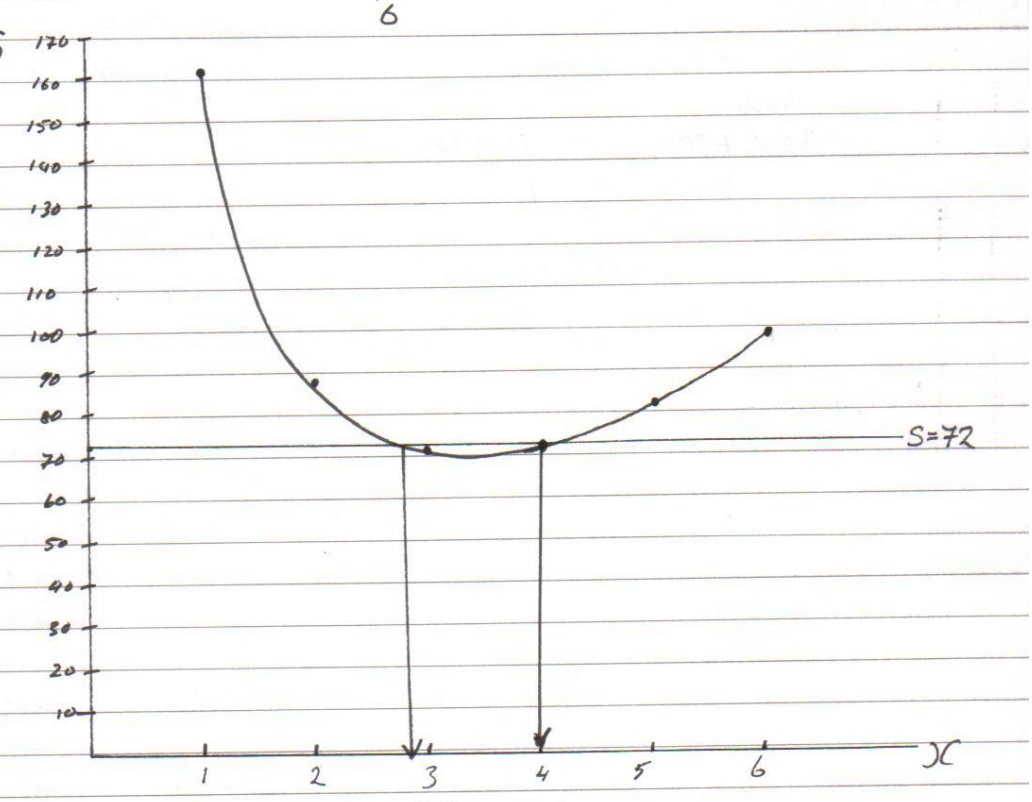
$x = 3$ $\quad S = 2(3)^2 + \frac{160}{3} \Rightarrow S = 71.3 \quad (3, 71.3)$

$x = 4$ $\quad S = 2(4)^2 + \frac{160}{4} \Rightarrow S = 72 \quad (4, 72)$ *

$x = 5$ $\quad S = 2(5)^2 + \frac{160}{5} \Rightarrow S = 82 \quad (5, 82)$

$x = 6$ $\quad S = 2(6)^2 + \frac{160}{6} \Rightarrow S = 98.6 \quad (6, 98.6)$

(iv)



$h = \frac{40}{x^2} \Rightarrow h = \frac{40}{4^2} \Rightarrow h = 2.5$ //

$h = \frac{40}{2.9^2} \Rightarrow h = 4.76$

Q4 Sells €11·50 Initial cost = €3500
+ €10·50 each game.

(i) $C(x) = 3500 + 10{\cdot}5x$

(ii) $I(x) = 11{\cdot}5x - 10{\cdot}5x \Rightarrow I(x) =$

Income = Sell − cost

$I(x) = 11{\cdot}5x - (3500 + 10{\cdot}5x)$

$I(x) = 11{\cdot}5x$

(iii) €

C: 500 games $\Rightarrow$ $C(x) = 3500 + 10{\cdot}5(500) = 8750$

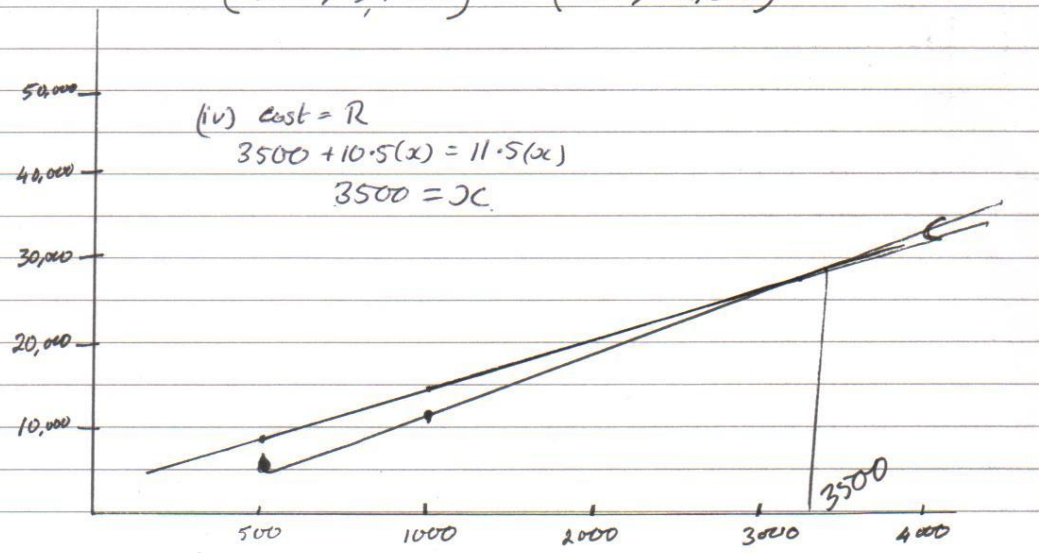
1000 games $\Rightarrow$ $C(x) = 3500 + 10{\cdot}5(1000) = 14000$

(500, 8750) (1000, 14,000)

I: 500 games $\Rightarrow$ $I(x) = 11{\cdot}5(500) = 5750$

1000 games $\Rightarrow$ $I(x) = 11{\cdot}5(1000) = 11500$

(500, 5,750) (1000, 11,500)



(iv) cost = R

$3500 + 10{\cdot}5(x) = 11{\cdot}5(x)$

$3500 = x$

(v) $P = I - C$ Profit.

(vi) ~~$5500 = 11.5x - (3500 + 10.5x)$~~

~~$5500 = 11.5x - 3500 - 10.5x$~~

~~$9000 = x$~~

$2000 = 11.5x - [3500 + 10.5x]$

$2000 = 11.5x - 3500 - 10.5x$

$5500 = x$

$4x + 7y = 165$

$3y = 219$

$y = 73$

$\times 1.20$

87.6

$\Rightarrow x = 23$

$x \cdot 80$

$+ \quad 18.40 \quad = 106.$

05

$4x + 7y = 96$

(i) $x + y = 96$

$\frac{x}{4} + \frac{y}{7} = 15 \quad (\times 28)$

$x + y = 96 \quad (\times 4)$

$7x + 4y = 420$

$28 \times 15 = 140 + 28 = 420$

$-4x + 4y = 384$

$7x + 4y = 420$

$3x = 36$

$x = 12 \Rightarrow y = 84$

$x \cdot 8 = 960$

$\times 1.2 = 168$

$+ \quad 84$

100.8

3:2

12.

2 | 3

(ii) 12

8 | 4 | 6

€ 110.40 =

Q6 (i) $C = 40x + 30{,}000$

(ii) $C = 40(6000) + 30{,}000 = 270000$

$\frac{270000}{6000} = 45$ per wheelbarrow

(iii) $\frac{40x + 30000}{x} = 46$

$40x + 30000 = 46x$

$30000 = 6x$

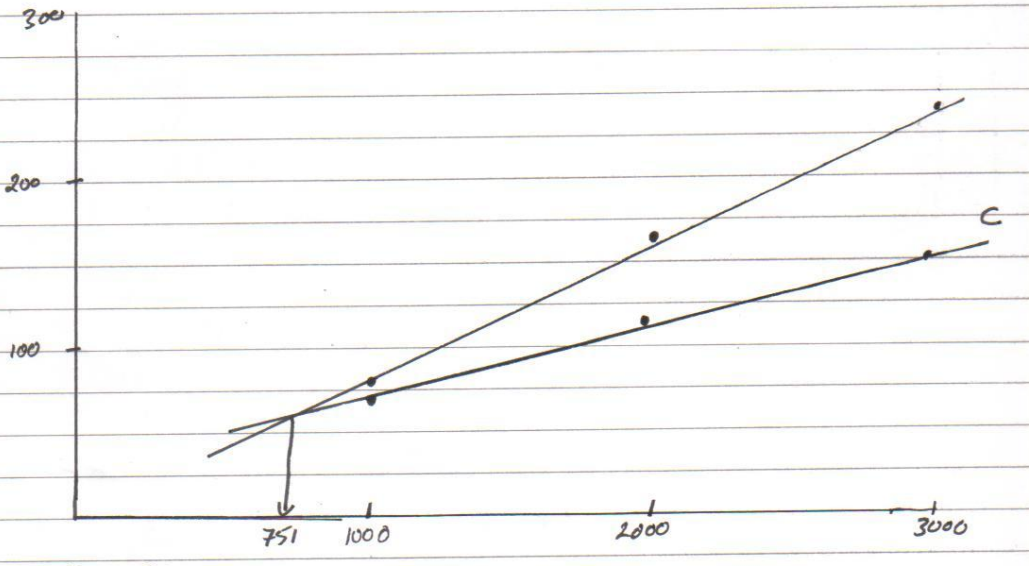
$5000 = x$

(iv) $R = 80x$

(v) $C = 40x + 30{,}000$ $\qquad R = 80x$

$x = 1000$	$C = 40000 + 30000 = 70{,}000$	$R = 80{,}000$
$x = 2000$	$C = 110{,}000$	$R = 160{,}000$
$x = 3000$	$C = 150{,}000$	$R = 240{,}000$

(vii) $P = 40x - 30{,}000$



(vi) $P = R - C$

$0 < 80x - 40x - 30{,}000$ $\qquad 30{,}000 < 40x$

$750 < x \Rightarrow 751.$