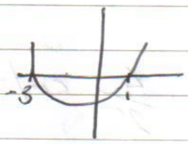
Revision Ex Core

Q4 $f(x) = x^3 + 3x^2 - 9x$

decreasing $\Rightarrow f'(x) < 0$

$$f'(x) = 3x^2 + 6x - 9 < 0$$
$$(\div 3) \quad x^2 + 2x - 3 = 0$$
$$(x+3)(x-1) = 0$$
$$x = -3 \quad x = 1$$



$f'(x) < 0$ for $-3 < x < 1$

Q7 $y = x \sin 2x$

slope $= \frac{dy}{dx} = (x)(\cos 2x)(2) + (\sin 2x)(1)$

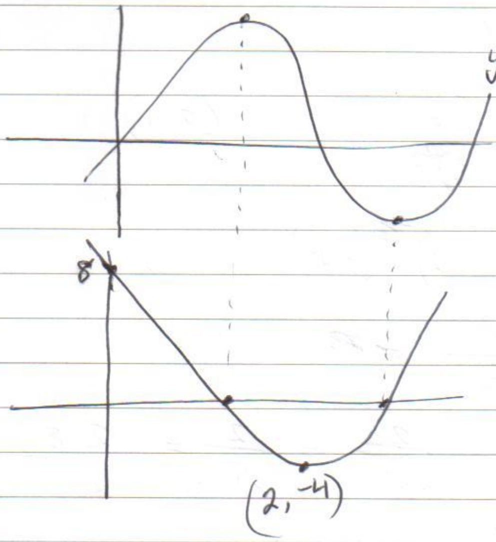
$$= 2x \cos x + \sin 2x$$

at $x = \pi/3$ $\quad 2\left(\frac{\pi}{3}\right) \cos 2\left(\frac{\pi}{3}\right) + \sin 2\left(\frac{2\pi}{3}\right)$

$$= \frac{2\pi}{3}\left(-\frac{1}{2}\right) + \sqrt{3}/2$$

$$= \sqrt{3}/2 - \pi/3$$

(c)



$$y = x^3 - 6x^2 + 8x$$

$$\frac{dy}{dx} = 3x^2 - 12x + 8$$

$$\frac{d^2y}{dx^2} = 6x - 12 = 0$$

$$6x = 12$$

$$x = 2$$

at $x = 2$

$$\frac{dy}{dx} = 3(2)^2 - 12(2) + 8$$

$$12 - 24 + 8$$

$$= -4.$$

$\Rightarrow$ Min Pt of $\frac{d^2y}{dx^2}$ is $(2, -4)$

Q7 $\quad y = xe^x$

$$\frac{dy}{dx} = (x)\,e^x + e^x(1)$$

$$xe^x + e^x = 0$$

$$e^x(x+1) = 0$$

$\cancel{e^x = 0}$ $\quad x = -1$ $\quad \Rightarrow y = (-1)e^{(-1)}$

$$= \frac{-1}{e}$$

$$\left(-1, \ -1/e\right)$$

$$\frac{dy}{dx} = e^x(x+1)$$

$$\frac{d^2y}{dx^2} = e^x(1) + (x+1)\,e^x$$

$$= e^x(1 + x + 1)$$

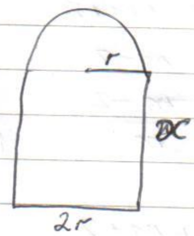
$$= e^x(x+2)$$

at $x = -1$ $\quad e^{-1}(-1+2) = e^{-1}(1) = \frac{1}{e} > 0 \Rightarrow$ Min

Q8

$A \to 2$
$B \to 4$
$C \to 1$
$D \to 3$

Q10



$P = 40$
$40 = \pi r + 2r + 2x$
$\Rightarrow 2x = 40 - \pi r - 2r$
$x = \frac{1}{2}(40 - \pi r - 2r)$

$$\text{Area} = \frac{1}{2}\pi r^2 + (2r)(x)$$
$$= \frac{1}{2}\pi r^2 + \cancel{2}r\left(\frac{1}{\cancel{2}}(40 - \pi r - 2r)\right)$$
$$= \frac{\pi r^2}{2} + 40r - \pi r^2 - 2r^2$$
$$A = 40r - 2r^2 - \frac{\pi r^2}{2}$$

Max $\Rightarrow \frac{dA}{dr} = 0$ $\qquad \frac{dA}{dr} = 40 - 4r - \pi r = 0$
$$4r + \pi r = 40$$
$$r(4 + \pi) = 40$$
$$r = \frac{40}{4 + \pi}$$

$$\left[\frac{d^2A}{dr^2} = -4 - \pi < 0 \Rightarrow \text{Max}\right]$$

$$\text{MAX Area} = 40\left(\frac{40}{4+\pi}\right) - 2\left(\frac{40}{4+\pi}\right)^2 - \frac{\pi}{2}\left(\frac{40}{4+\pi}\right)^2 = 112 \text{ sq units}$$

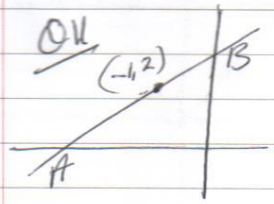
$$\frac{1600}{4+\pi} - \frac{3200}{(4+\pi)^2} - \frac{\pi}{\cancel{2}}\left(\frac{\cancel{1600}^{800}}{(4+\pi)^2}\right)$$

$$\frac{1600(4+\pi) - 3200 - 800\pi}{(4+\pi)^2}$$

$$= \frac{6400 + 1600\pi - 3200 - 800\pi}{(4+\pi)^2} = \frac{3200 + 800\pi}{(4+\pi)^2} = \frac{800(4+\pi)}{(4+\pi)^2}$$

$$\boxed{= \frac{800}{4+\pi}}$$

Q11



$$y-2 = m(x+1)$$
$$y-2 = mx + m.$$
$$y = mx + m + 2$$

cuts $x \Rightarrow y = 0$ $\qquad 0 = mx + m + 2$
$$mx = -m-2$$
$$x = \frac{-m-2}{m} \qquad A\left(\frac{-m-2}{m}, 0\right)$$

cuts $y \Rightarrow x = 0$ $\qquad y = m(0) + m + 2$
$$y = m+2 \qquad B(0, m+2)$$

(i) Area Δ $= \frac{1}{2}$ base $\times h$
$$= \frac{1}{2}\left(\frac{+m+2}{m}\right)(m+2)$$
$$= \frac{(m+2)^2}{2m}$$

(ii) Min $A \Rightarrow \frac{dA}{dm} = 0.$

$$\frac{dA}{dm} = \frac{(2m)2(m+2)(1) - (m+2)^2(2)}{(2m)^2} = 0$$

$$4m^2 + 8m - 2m^2 - 8m - 8 = 0$$
$$2m^2 - 8 = 0$$
$$m^2 - 4 = 0$$
$$(m+2)(m-2) = 0$$

slope is Not neg! $\Rightarrow$ $\cancel{m = -2}$ $\qquad \underline{\underline{m = 2}}$

$$\therefore A = \frac{(2+2)^2}{2(2)} = \frac{16}{4} = 4 \text{ sq units.}$$

Q13 $f(x) = x^3 + 3kx^2 + 32$

$f'(x) = 3x^2 + 6kx = 0$

$3x(x + 2k) = 0$

$x = 0 \qquad x = -2k.$

find y co-ords

at $x=0$ $\quad y = (0)^3 + 3k(0)^2 + 32 = 32 \qquad (0, 32)$

at $x = -2k$ $\quad y = (-2k)^3 + 3k(-2k)^2 + 32$

$= -8k^3 + 12k^3 + 32$

$= 4k^3 + 32 \qquad (-2k, 4k^3+32)$

$f(x) = 0$ 3 Roots, 2 equal roots.

$\Rightarrow (-2k, 4k^3+32)$ is on x axis.

$\Rightarrow 4k^3 + 32 = 0$

$4k^3 = -32$

$k^3 = -8$

$k = \sqrt[3]{-8} = -2.$

RevEx (Extended-Response)

Q3 $h = 2 + 40t - 5t^2$

(i) $\frac{dh}{dt} = 40 - 10t$

(a) at $t = 2$ $\quad dh/dt = 40 - 10(2)$
$= 20 m/s.$

(b) at $t = 2.5$ $\quad \frac{dh}{dt} = 40 - 10(2.5)$
$= 15 m/s$

(ii) $dh/dt = 0$ $\quad 40 - 10t = 0$
$10t = 40$
$t = 4$ sec

The ball has stopped.

(iii) h at $t = 4$.
$h = 2 + 40(4) - 5(4)^2$
$= 2 + 160 - 80$
$= 82m$

(iv) at $t = 6$ $\quad dh/dt = 40 - 10(6)$
$= -20$

The ball is falling back down. at 20m/sec

(v) speed $\left(\frac{dh}{dt}\right)$ when $t = 0$
$40 - 10(0) = 40m/sec$

(vi) Ball hits ground $\Rightarrow h = 0$.
$2 + 40t - 5t^2 = 0$
$5t^2 - 40t - 2 = 0$

$t = \frac{40 \pm \sqrt{40^2 - 4(5)(-2)}}{2(5)} = \frac{40 \pm \sqrt{1640}}{10}$ $\nearrow t = 8.0497$ $\searrow t = \cancel{-0.0497}$

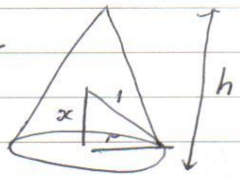
$t = 8.05$

Speed $\left(\frac{dh}{dt}\right)$ at $t = 8.05$.

$$40 - 10(8.05)$$
$$= -40.5$$

$\Rightarrow$ Speed is $40.5\,m/sec$ when hits the ground.

Q4



(a) $V_{cone} = \frac{1}{3}\pi r^2 h$

(i) $1^2 = x^2 + r^2$

$\sqrt{1-x^2} = r$

(ii) $h = x + 1$

(B) $V = \frac{1}{3}\pi r^2 h$

$$V = \frac{1}{3}\pi\left(\sqrt{1-x^2}\right)^2(x+1)$$
$$= \frac{1}{3}\pi(1-x^2)(x+1)$$
$$= \frac{\pi}{3}(x + 1 - x^3 - x^2)$$
$$= \frac{\pi}{3}(1 + x - x^2 - x^3).$$

(c) x cannot be greater than hypotenuse, 1

$\Rightarrow$ domain $0 < x < 1$

(d) (i) $\frac{dV}{dx} = \frac{\pi}{3}(1 - 2x - 3x^2) + (1 + x - x^2 - x^3)(0)$

$$= \frac{\pi}{3}(1 - 2x - 3x^2)$$

(ii) $\frac{\pi}{3}\left(1-2x-3x^2\right)=0$

$$1-2x-3x^2=0$$
$$3x^2+2x-1=0$$
$$(3x-1)(x+1)=0$$
$$x=\tfrac{1}{3} \quad \cancel{x=-1}$$

$$\Rightarrow \quad x=\tfrac{1}{3}$$

(iii) $\frac{d^2V}{dx^2} = \frac{\pi}{3}\left(-2-6x\right)$

at $x=\tfrac{1}{3}$ $\quad \frac{\pi}{3}\left(-2-\overset{2}{\cancel{6}}\left(\tfrac{1}{3}\right)\right)$

$$= \frac{\pi}{3}(-4) = \frac{-4\pi}{3} < 0 \Rightarrow \text{Max}$$

$$\therefore \text{Max } V = \frac{\pi}{3}\left(1+\frac{1}{3}-\left(\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^3\right)$$

$$= \frac{\pi}{3}\left(1+\frac{1}{3}-\frac{1}{9}-\frac{1}{27}\right)$$

$$= \frac{32\pi}{81}\ m^3$$

Q6 $P = 10 + 40r - 20r^2$

(a) centre $\Rightarrow r = 0$

$\therefore P = 10 \Rightarrow$ Population $= 10,000$.

(b) Domain of r: P cannot fall below zero

$\Rightarrow$ solve $0 = 10 + 40r - 20r^2$

$20r^2 - 40r - 10 = 0$

$2r^2 - 4r - 1 = 0$

~~(2)( )~~

$$r = \frac{4 \pm \sqrt{(4)^2 - 4(2)(-1)}}{2(2)}$$

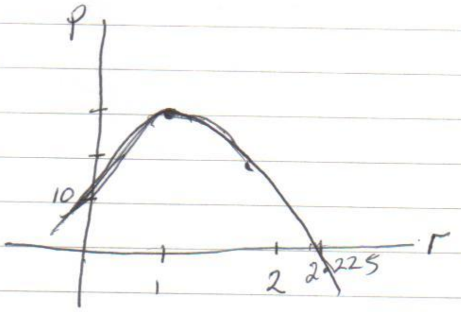
$$= \frac{4 \pm \sqrt{24}}{4}$$

$$\frac{4 \pm 2\sqrt{6}}{4}$$

$$= 1 \pm \tfrac{1}{2}\sqrt{6}.$$

$\Rightarrow$ Domain is $0 < r < \left(1 + \frac{\sqrt{6}}{2}\right)$ (2.225)

(c)



at $r=0$ $P = 10$.

Domain $0 < r < 2.225$.

Max pt

$\frac{dP}{dr} = 0$

$40 - 40r = 0$

$40r = 40$

$r = 1$.

at $r=1$ $P = 10 + 40(1) - 20(1)^2$

$P = 30$.

(d) $\frac{dP}{dr} = 40 - 40r$

(e) $\frac{dP}{dr} = 40 - 40r$

at $r = 0.5$ $\quad 40 - 40(0.5) = 20$

at $r = 1$ $\quad 40 - 40(1) = 0$

at $r = 2$ $\quad 40 - 40(2) = -40.$

(f) greatest $\Rightarrow$ Max $\Rightarrow \frac{dP}{dr} = 0.$

$$40 - 40r = 0$$
$$40r = 40$$
$$r = 1.$$

Population ; $\quad P = 10 + 40(1) - 20(1)^2$

$= 30$

$\Rightarrow$ population $= 30,000.$

Q7 $\quad h = \left(10 - \frac{t}{200}\right)^2$

(a) h at $t=0$.

$$h = \left(10 - \frac{0}{200}\right)^2 = 100\text{ cm}$$

(b) $h = 64$.

$$64 = \left(10 - \frac{t}{200}\right)^2$$

$$8 = 10 - t/200$$

$$\frac{t}{200} = 2$$

$$t = 400\text{ sec}$$

(c) $\frac{dV}{dt}$ when $h = 24$. and $r = 52$.

$$V = \pi r^2 h. = \pi (52)^2 h = 2704\pi h.$$

$$\frac{dV}{dh} = 2704\pi$$

$$h = \left(10 - \frac{t}{200}\right)^2$$

$$\frac{dh}{dt} = 2\left(10 - t/200\right)\left(\frac{-1}{200}\right) = \frac{-1}{100}\left(10 - t/200\right)$$

$$= \frac{-10}{100} + \frac{t}{20000}$$

$$= -1/10 + t/20000$$

When $h = 24$ $\quad t = 400$ (from part b)

$$\Rightarrow \frac{-1}{10} + \frac{400}{20000} = -\frac{2}{25}$$

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$$= (2704\pi)\left(\frac{-2}{25}\right) = -\frac{5408}{25}\pi = -679.59$$

$$= -680\text{ cm}^3/\text{s}$$

$\Rightarrow$ ~~Rate~~ Vol water decreasing at 680 cm^3 per sec.

(d) area of hole, circle $= \pi r^2$ $r = 1$

$\Rightarrow A = \pi$.

$\frac{dV}{dt} = \text{Area} \times \text{Speed}$.

$680 = \pi \times \text{Speed}$

$\cancel{\frac{680}{\pi} = \text{Speed}}$

$\cancel{216 \cdot 45 = \text{Speed}}$.

$\frac{5408}{25}\cancel{\pi} = \cancel{\pi} \times \text{Speed}$.

$\underline{216 \cdot 32 = \text{Speed}}$

(e)

$$h = \left(10 - \frac{t}{200}\right)^2$$

$$\sqrt{h} = 10 - t/200$$

Speed of Water = rate at which Vol decreasing $\div$ area (π)

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$$\cancel{= (2704\pi)\left(-\frac{1}{10} + \frac{t}{20000}\right)}$$

$$\cancel{= -270 \cdot 4\pi + \frac{2704t\pi}{20000}}$$

$$\frac{2704\pi\left(\frac{-1}{100}\left(10 - \frac{t}{200}\right)\right)}{\pi}$$

$$= 2704\left(-\frac{1}{100}\left(\sqrt{h}\right)\right)$$

$$= -27 \cdot 04\sqrt{h}.$$

(f) $V = C\sqrt{1962h}$

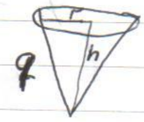
$$27{\cdot}04\sqrt{h} = C\sqrt{1962}\sqrt{h}$$

$$\frac{27{\cdot}04}{\sqrt{1962}} = C$$

$$0{\cdot}61 = C$$

$$\underline{\underline{0{\cdot}6 = C}}$$

Q9



Slant height = 9cm (radius of circle)

$$9^2 = r^2 + h^2$$

$$81 - h^2 = r^2$$

(a) Vol cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(81-h^2)h$$

$$= \frac{\pi}{3}h(81-h^2)$$

(b) Vol $= \frac{154\pi}{3}$ $\qquad \frac{154\pi}{3} = \frac{\pi}{3}h(81-h^2)$

$$154 = 81h - h^3$$

$$h^3 - 81h + 154 = 0$$

Try $h = 2$ $\qquad (2)^3 - 81(2) + 154 = 0$

$$0 = 0 \checkmark$$

$$\Rightarrow h = 2.$$

$$\begin{array}{r} h^2 + 2h - 77 \\ h-2 \,\big)\, h^3 - 81h + 154 \\ \ominus\, h^3 \oplus 2h^2 \\ \hline 2h^2 - 81h + 154 \\ \ominus\, 2h^2 \oplus 4h \\ \hline -77h + 154 \\ \oplus\, -77h \ominus 154 \\ \hline \end{array}$$

$$h^2 + 2h - 77 = 0$$

$$\frac{-2 \pm \sqrt{(2)^2 - 4(1)(-77)}}{2(1)}$$

$$\frac{-2 \pm \sqrt{312}}{2} \quad \nearrow 7{\cdot}831 \quad \searrow -9{\cdot}831$$

Non integer value $= 7{\cdot}83$

(c) Max Vol $\Rightarrow \frac{dv}{dh} = 0$.

$$V = \frac{\pi}{3}h(81 - h^2)$$
$$= 27\pi h - \frac{\pi}{3}h^3$$
$$\frac{dV}{dh} = 27\pi - \pi h^2 = 0$$
$$27 - h^2 = 0$$
$$h^2 = 27$$
$$h = \sqrt{27} = 3\sqrt{3}$$
$$h = 5.2.$$

$$\frac{d^2V}{dh^2} = -2\pi h$$

at $h = 5.2 \Rightarrow -2\pi(5.2) = -10.4\pi < 0 \Rightarrow$ Max

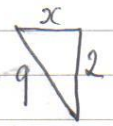
$$Vol = \frac{\pi}{3}h(81 - h^2)$$

at $h = 3\sqrt{3}$ $\quad Vol = \frac{\pi}{\cancel{3}}(\cancel{3}\sqrt{3})\left(81 - (3\sqrt{3})^2\right)$

$$= \sqrt{3}\pi(54)$$
$$= 294\,cm^3$$

(d)

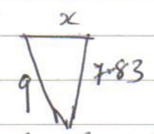
	Cups in (b)		Cups in (c)
radius	8.77	4.44.	7.35.
height	2	7.83	5.2
Capacity	$\frac{154\pi}{3} = 161$	$\frac{154\pi}{3} = 161$	294



$9^2 = x^2 + 2^2$

$\sqrt{81-4} = x$

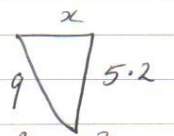
$8.77 = x$



$9^2 = x^2 + 7.83^2$

$\sqrt{9^2 - 7.83^2} = x$

$4.44 = x$



$9^2 = x^2 + 5.2^2$

$\sqrt{9^2 - 5.2^2} = x$

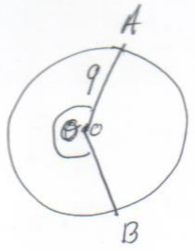
$7.35 = x$

(e)

$r = 4.44$ and $h = 7.83$

as will be well proportioned and easy to handle.

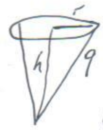
(f)



$r = 4.44$

$h = 7.83$

$V = 161$



$CSA = \pi r l$

$= \pi(4.44)(9)$

$= 125.538 cm^2$

area of sector $= 125.538$

$\frac{\theta}{360} \pi r^2 = 125.538$

$\frac{\theta}{360} \pi 9^2 = 125.538$

$\theta = \frac{125.538 \times 360}{81\pi}$

$\theta = 177.6$

$\theta = 178^{\circ}$.