Rev Ex (Core)

Instalment Savings

Q2 300 a month for 8 yrs ⇒ 96 months
6% per annum ⇒ 0.06

Monthly $(1+r)^{12} = 1.06$

$r = 1.06^{1/12} - 1 = 0.004867$

$FV = 300(1.004867) + 300(1.004867)^2 + \cdots 300(1.004867)^{96}$

$S_n = \frac{a(1-r^n)}{1-r}$ $\quad a = 300(1.004867)$

$r = 1.004867$

$n = 96$

$FV = \frac{300(1.004867)\left[1-(1.004867)^{96}\right]}{1-1.004867}$

$FV = 36777.53$

Q3 Loan 20 000 25 payments.

2% = 0.02

$A = \frac{P\,i(1+i)^t}{(1+i)^t - 1}$

$A = \frac{20{,}000(0.02)(1.02)^{25}}{((1.02)^{25}-1)}$

$A = 1024.41$

$= 1024$ to nearest euro

Q5

1.25% per month $\Rightarrow 0.0125$

$(1+r)^{12} = 1+i$

$(1+0.0125)^{12} = 1+i$

$(1.0125)^{12} - 1 = i$

$0.16075 = i$

$\Rightarrow$ annual Rate = 16%

2.5% Per Month $\Rightarrow 0.025$

$(1+r)^{12} = 1+i$

$(1+0.025)^{12} = 1+i$

$(1.025)^{12} - 1 = i$

$0.3448 = i$

$\Rightarrow$ Annual Rate = 34.5%

Q8 3000 per yr at 7·3% for 8 yrs
$\Rightarrow 0.073$

(i) $P = \frac{F}{(1+i)^t}$ instalments!

$\Rightarrow P = \frac{3000}{} + \frac{3000}{(1.073)} + \frac{3000}{(1.073)^2} + \cdots \frac{3000}{(1.073)^7}$

$S_n = \frac{a(1-r^n)}{1-r}$ $\quad a = 3000$

$r = \frac{1}{1.073}$

$n = 8$

$PV = \frac{(3000)\left[1-\left(\frac{1}{1.073}\right)^8\right]}{\left(1-\frac{1}{1.073}\right)}$

$= 19000.13$

(ii) $F = P(1+i)^t$

$= 19000.13(1+0.073)^8$

$= 33385.23$

(iii) $FV = 3000(1.073) + 3000(1.073)^2 + \cdots 3000(1.073)^8$

$a = 3000(1.073) \quad r = 1.073 \quad n = 8$

$FV = \frac{3000(1.073)(1-(1.073)^8)}{1-1.073} = 33385.23$

True ✓

Rev Ex: Advanced

Q1 500000 in 10 yrs. annual rate 9% = 0.09.

(i) Present Value $= \frac{F}{(1+i)^t}$

$= \frac{500000}{(1.09)^{10}}$

$= 211{,}205.40$

(ii) $FV = P + P(1+.09) + P(1+.09)^2 + \cdots P(1+.09)^{10}$

$a = P$, $r = 1.09$, $n = 10$

$500{,}000 = P\left[\frac{1-(1.09)^{10}}{1-1.09}\right]$

$500{,}000 = P(15.1929)$

$32910.04 = P$

Q4 Mortgage: $A = P\left[\frac{i(1+i)^t}{(1+i)^t-1}\right]$

$10\% = 0.1$ 20 yrs. = 240 months

$(1+r)^{12} = 1.1$

$r = 1.1^{1/12} - 1$

$r = 0.007941$

$700 = \frac{P(0.007941)(1+0.007941)^{240}}{(1+0.007941)^{240}-1}$

$700 = P(0.00934\cdots)$

$74943 = P$

Q2 35 yrs old → 65 yrs old = 30 yrs.

65 yrs → 100 = 35 yrs of pension.

4% = 0.04 300,000 to Move.

20,000 pension per yr.

(i) Pension:

$$PV = 20000 + \frac{20000}{1.04} + \frac{20000}{(1.04)^2} + \cdots \frac{20000}{(1.04)^{35}}$$

$$S_n = \frac{a(1-r^n)}{1-r} \qquad a = 20{,}000$$

$$r = \frac{1}{1.04}$$

$$n = 35$$

$$PV = \frac{\frac{20{,}000}{1.04}\left[1-\left(\frac{1}{1.04}\right)^{35}\right]}{\left(1-\frac{1}{1.04}\right)}$$

$$= 373292.26.$$

Total Required: 373292.26
+ 300000
673,292.26.

(ii) 40,000 at 5% (⇒ 0.05).

$$F = P(1+i)^t$$
$$= 40{,}000(1.05)^{30}$$
$$= 172877.69$$

⇒ 673292.26 − 172877.69
= 500414.57 to be saved.

30 yrs at 5%.

$$500414.57 = P(1.05) + P(1.05)^2 + \cdots P(1.05)^{30}$$

$a = P(1.05)$

$r = 1.05$

$n = 30$

$$500414.57 = \frac{P(1.05)\left[1-(1.05)^{30}\right]}{(1-1.05)}$$

$$500414.57 = P(69.7607\ldots)$$

$$7173.30 = P$$

$\Rightarrow$ 7,173.30 to be invested each yr.

(iii) $\Rightarrow$ 673292.26 to be saved in 25 yrs at 5%

$$673292.26 = P(1.05) + P(1.05)^2 + \cdots P(1.05)^{25}$$

$$673292.26 = \frac{P(1.05)\left[1-(1.05)^{25}\right]}{(1-1.05)}$$

$$673292.26 = P(50.11345\ldots)$$

$$13435.36 = P$$

$\Rightarrow$ 13,435.36 to be ~~saved~~ invested each yr.

Q3 Mortgage €100,000 9% → 0.09

€800 per month.

(i)

$$A = \frac{P\,i\,(1+i)^t}{(1+i)^t - 1}$$

A = Repayments P = Mortgage Value.
i = interest rate t = time months/yrs

(ii) Annual = 9%

Monthly

$$(1+r)^{12} = 1.09$$
$$r = 1.09^{1/12} - 1$$
$$r = 0.007207.$$

(iii)

$$800 = 100000\,\frac{(0.007207)(1+0.007207)^t}{(1+0.007207)^t - 1}$$

$$800 = \frac{720.7\,(1.007207)^t}{(1.007207)^t - 1}$$

$$800\left[(1.007207)^t - 1\right] = 720.7\,(1.007207)^t$$

$$\frac{800}{720.7}\left[(1.007207)^t - 1\right] = (1.007207)^t$$

$$1.11\left[(1.007207)^t - 1\right] = (1.007207)^t$$

$$1.11(1.007207)^t - 1.11 = (1.007207)^t$$

$$1.11(1.007207)^t - (1.007207)^t = 1.11$$

$$(1.007207)^t = \frac{1.11}{.11}$$

$$(1.007207)^t = 10.090909.$$

$$t \log 1.007207 = \log 10.090909.$$

$$t = \frac{\log 10.090909}{\log 1.007207}$$

$$t = 321.903 \text{ months.}$$

(iv) 322 months = 26 yrs 10 months.