Solutions to deferred material Text and Tests 5

Inferential Statistics (Chapter 5)

Test yourself 5

A questions

1. $\mu = 25\ kg,\ \sigma = \sqrt{5}\ kg$ and $n = 50$

From 24.5kg to 25.5kg

For $\bar{x} = 24.5$, standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{24.5-25}{\frac{\sqrt{5}}{\sqrt{50}}} = \frac{-0.5}{0.31622} = -1.58$

For $\bar{x} = 25.5$, standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{25.5-25}{\frac{\sqrt{5}}{\sqrt{50}}} = \frac{0.5}{0.31622} = 1.58$

$$P(24.5 \le \bar{x} \le 25.5) = P(-0.1.58 \le z \le 1.58)$$
$$= P(z \le 1.58) - P(z > 1.58)$$
$$= P(z \le 1.58) - [1 - P(z \le 1.58)]$$
$$= 0.9394 - [1 - 0.9394]$$
$$= 0.8788$$
$$= 0.89$$

2. $\mu = 2.85,\ \sigma = 0.07$ and $n = 20$

Mean of sample $\bar{x} = \mu = 2.85$

Standard error on the mean $= \frac{\sigma}{\sqrt{n}} = \frac{0.07}{\sqrt{20}} = 0.016$

3. $\bar{x} = 26.2\ beats$ and $\sigma_{\bar{x}} = 5.15\ beats,\ n = 32$

The 95% confidence interval [CI] for μ is $\bar{x} \pm 1.96 \frac{\sigma_{\bar{x}}}{\sqrt{n}}$

$$\therefore CI = 26.2 \pm 1.96 \frac{5.15}{\sqrt{32}}$$
$$= 26.2 \pm 1.7843$$
$$= 24.4157,\ 27.9843$$
$$\Rightarrow 24.42\ beats < \mu < 27.98\ beats$$

4. $\bar{x} = 266ml,\ \sigma = 20ml$ and $n = 40$

The 95% confidence interval [CI] for μ is $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

$$\therefore CI = 266 \pm 1.96 \frac{20}{\sqrt{40}}$$
$$= 266 \pm 6.198$$
$$= 259.802,\ 272.198$$
$$\Rightarrow 259.80ml < \mu < 272.20ml$$

5. $n = 150$, the sample proportion $\hat{p} = \frac{90}{150} = 0.6$

The 95% confidence interval [CI] for a proportion $p = \hat{p} \pm 1.96\sqrt{\frac{p(1-p)}{n}}$

$$\therefore CI = 0.6 \pm 1.96\sqrt{\frac{0.6(1-0.6)}{150}}$$

$$= 0.6 \pm 1.96(0.04)$$

$$= 0.6 \pm 0.0784$$

$$= 0.5216,\ 0.6784$$

$$\Rightarrow 0.522 < p < 0.678$$

6. $n = 100$, the sample proportion $\hat{p} = 0.55$

The 95% confidence interval [CI] for a proportion $p = \hat{p} \pm 1.96\sqrt{\frac{p(1-p)}{n}}$

$$\therefore CI = 0.55 \pm 1.96\sqrt{\frac{0.55(1-0.55)}{100}}$$

$$= 0.55 \pm 1.96(0.04975)$$

$$= 0.55 \pm 0.09750$$

$$= 0.4525,\ 0.6475$$

$$\Rightarrow 0.453 < p < 0.648$$

7. (i) H_0: the height of the Irish students does not differ from the height of the German students

H_1: the height of the Irish students does differ from the height of the German students

(ii) 5% level of significance $\Rightarrow -1.96 < z < 1.96$

(iii) $\mu = 176cm$, $\bar{x} = 179cm, \sigma = 11cm$ and $n = 60$

(iv) Standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{179-176}{\frac{11}{\sqrt{60}}} = \frac{3}{1.42} = 2.11$

Since 2.11 > 1.96, the test statistic is in the critical region and hence we reject the null hypothesis that the heights are the same

Yes there is evidence to suggest that the average German student is taller than the average Irish student.

8. (i) H_0: the mean quantity of honey has not changed

H_1: the mean quantity of honey has changed

(ii) $\mu = 460.3g$, $\bar{x} = 461.2g, \sigma = 3.2g$ and $n = 60$

Standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{461.2-460.3}{\frac{3.2}{\sqrt{60}}} = \frac{0.9}{0.4131} = 2.18$

(iii) 5% level of significance $\Rightarrow -1.96 < z < 1.96$

Since $2.18 > 1.96$, the test statistic is in the critical region and hence we reject the null hypothesis that the quantity of honey has not changed

Yes there is evidence to suggest that the sample mean is different from the population mean.

9. (i) A Normal distribution. Central Limit Theorem

(ii) Because the sample size $n > 30$

(iii) $\mu = 96\ hrs$, $\sigma = 6\ hrs\ and\ n = 36$

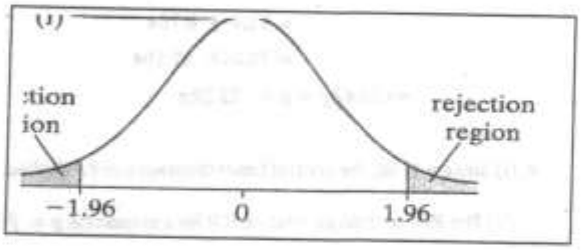
Greater than 98 hours $\Longrightarrow$ $\bar{x} > 98$ hours

Standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{98-96}{\frac{6}{\sqrt{36}}} = \frac{2}{1} = 2$

$$P(\bar{x} > 98) = P(z > 2)$$
$$= 1 - P(z \leq 2)$$
$$= 1 - 0.9772 = 0.0228$$
$$P(\bar{x} > 98)(40) = 0.0228(40) = 0.912 = 1$$

10.

(i)



(ii) $z < -1.96, z > 1.96$

(iii) The sample statistic is $z_1 = 1.6$

(ii) $p - value = 2 \times P(z > |z_1|)$

$$= 2 \times P(z > |1.6|)$$
$$= 2 \times [1 - P(z \leq |1.6|)$$
$$= 2 \times [1 - 0.9452)$$
$$= 0.1096$$

Test yourself 5

B questions

1. $\mu = 74$ and $\sigma = 6$, $P(\bar{x} > 72) = 0.854$

If $P(\bar{x} > 72) = 0.854 \Longrightarrow z = 1.05$ using page 37 of the tables.

$$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = 1.05$$

$$\therefore 72 - 74 = 1.05\left(\frac{6}{\sqrt{n}}\right)$$

$$-2\sqrt{n} = 1.05(6)$$

(Squaring both sides) $4n = 39.69$

$$n = 9.92 = 10$$

2. $\mu = 12.1kg$ and $\sigma = 0.4kg$

For $x = 12$, standard unit $z = \frac{x-\mu}{\sigma} = \frac{12-12.1}{0.4} = \frac{-0.1}{0.4} = -0.25$

$$P(x \leq 12kg) = P(z \leq -0.25) = [1 - P(z \leq 0.25)]$$

$$= 1 - 0.5987$$

$$= 0.401$$

3. $\bar{x} = 31.4kg$ and $\sigma = 2.4kg$, $n = 36$

The 95% confidence interval [CI] for μ is $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$

$$\therefore CI = 31.4 \pm 1.96\frac{2.4}{\sqrt{36}}$$

$$= 31.4 \pm 0.784$$

$$= 30.616,\ 32.184$$

$$\Rightarrow 30.6kg < \mu < 32.2kg$$

4. (i) Since $n \geq 30$, the central Limit theorem can be applied.

(ii) The 95% confidence interval [CI] for a proportion $p = \hat{p} \pm 1.96\sqrt{\frac{p(1-p)}{n}}$

$p = 20\% = 0.2$

$$\therefore CI = 0.2 \pm 1.96\sqrt{\frac{0.2(1-0.2)}{30}}$$

$$= 0.2 \pm 1.96(0.073)$$

$$= 0.2 \pm 0.1431$$

$$= 0.0569,\ 0.3431$$

$$\Rightarrow 0.057 < p < 0.343$$

$$\Rightarrow 5.7\% < p < 34.3\%$$

5. (i) If 100 samples of the same size are taken, then the true population mean (or proportion) will lie in the given interval on 95 occasions out of 100.

(ii) $\bar{x} = 13.52km/l$ and $\sigma = 2.23km/l$, $n = 150$

The 95% confidence interval [CI] for μ is $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$

$$\therefore CI = 13.52 \pm 1.96\frac{2.23}{\sqrt{150}}$$

$$= 13.52 \pm 0.35687$$

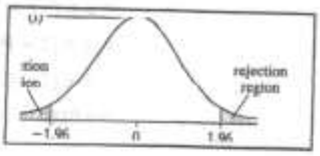
$= 13.163,\ 13.877$

$\Rightarrow 13.16km/l < \mu < 13.88km/l$

6. (i) H_0: the mean response time is unchanged, $\mu = 1.2s$

H_1: the mean response time changes, $\mu \neq 1.2s$

(ii) $z < -1.96, z > 1.96$



$\mu = 1.2s, \bar{x} = 1.05s$ and $\sigma = 0.5s,\ n = 100$

(iii) The test statistic is $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1.05-1.2}{\frac{0.5}{\sqrt{100}}} = \frac{-0.15}{0.05} = -3$

Since $-3 < -1.96$, the test statistic is in the critical region and hence we reject the null hypothesis that the mean response time is 1.2s

Yes the drug has an effect on the response time.

(iv) $p - value = 2 \times P(z > |z_1|)$

$= 2 \times P(z > |-3|)$

$= 2 \times [1 - P(z \leq |3|)$

$= 2 \times [1 - 0.9987)$

$= 0.0026$

Since $p < 0.05$ we reject the null hypothesis that $\mu = 1.2s$

7. $n = 72$, the sample proportion $\hat{p} = \frac{50}{72} = 0.6944$

(i) margin of error $= \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{72}} = 0.118$

(ii) $n = 72$, the sample proportion $\hat{p} = \frac{50}{72} = 0.69$

(iii) The 95% confidence interval [CI] for a proportion $p = \hat{p} \pm \frac{1}{\sqrt{n}}$

$p = \frac{50}{72} \pm \frac{1}{\sqrt{72}}$

$0.577 < p < 0.812$

(iv) Since 80% = 0.8 is within the confidence interval we accept the school's claim.

8. (i) H_0: the mean weight has not changed, $\mu = 25kg$

H_1: the mean weight has changed, $\mu \neq 25kg$

(ii) $\mu = 25kg$, $\bar{x} = 24.5kg, \sigma = 1.5kg$ and $n = 50$

Sample statistic, $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{24.5-25}{\frac{1.8}{\sqrt{80}}} = \frac{-0.5}{0.212} = -2.36$

(iii) $p-value = 2 \times P(z > |z_1|)$

$= 2 \times P(z > |-2.36|)$

$= 2 \times [1 - P(z \leq |2.36|)$

$= 2 \times [1 - 0.9909)$

$= 0.0182$

(iv) At the 5% level of significance since $p < 0.05$ we reject the null hypothesis that $\mu = 25kg$, the wholesaler's suspicion is justified.

(v) The $p-value$ is the smallest level of significance at which the null hypothesis could be rejected.

9. (i) Mean = 68kg,

Standard error = $\frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = 0.65kg$

(ii) Standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{67.5-68}{\frac{3}{\sqrt{25}}} = \frac{-0.5}{0.65} = -0.769$

$P(\bar{x} < 667.5) = P(z < -0.769)$

$= 1 - P(z \leq 0.77)$

$= 1 - 0.7794 = 0.2206$

No of samples = 0.2206(80) = 17.64 = 17

Test yourself 5

C questions

1. (i) H_0: the mean weight has not changed, $\mu = 500g$

H_1: the mean weight has changed, $\mu \neq 500g$

(ii) $\mu = 500g$, $\bar{x} = 505g, \sigma = 18g$ and $n = 36$

Sample statistic, $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{505-500}{\frac{18}{\sqrt{36}}} = \frac{5}{3} = -1.67$

(iii) $p-value = 2 \times P(z > |z_1|)$

$= 2 \times P(z > |-16.7|)$

$= 2 \times [1 - P(z \leq |1.67|)$

$= 2 \times [1 - 0.9525)$

$= 0.095$

(iv) Since $p > 0.05$ we accept the null hypothesis that $\mu = 500g$. The result is not significant, the mean weight has not changed.

2. (i) $n = 80$, the sample proportion $\hat{p} = \frac{28}{80} = 0.35$

The standard error $= \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.35(1-0.35)}{80}}$

$= 0.0533$

(ii) The 95% confidence interval [CI] for a proportion $p = \hat{p} \pm 1.96\sqrt{\frac{p(1-p)}{n}}$

$\therefore CI = 0.35 \pm 1.96\sqrt{\frac{0.35(1-0.35)}{80}}$

$= 0.35 \pm 1.96(0.0533)$

$= 0.35 \pm 0.1045$

$= 0.245,\ 0.455$

$\Rightarrow 0.245 < p < 0.455$

3. (i) Mean weight $= \bar{x} = \frac{79.93+82.87}{2} = 81.4g$

(ii) The 95% confidence interval [CI] for μ is $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$

$\Rightarrow \bar{x} + 1.96\frac{\sigma}{\sqrt{n}} = 82.87$ *and* $\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} = 79.93$

$\Rightarrow 81.4 + 1.96\frac{\sigma}{\sqrt{400}} = 82.87$

$\Rightarrow \quad 1.96\frac{\sigma}{20} = 1.47$

$\sigma = 15g$

4. (i) $x < 475g, \mu = 500g, \sigma = 20g$

Standard unit $z = \frac{\bar{x}-\mu}{\sigma} = \frac{475-500}{20} = \frac{-25}{20} = -1.25$

$P(x < 475) = P(z < -1.25)$

$= 1 - P(z \leq 1.25)$

$= 1 - 0.8944 = 0.1056$

$= 10.6\%$

$x > 530g, \mu = 500g, \sigma = 20g$

Standard unit $z = \frac{\bar{x}-\mu}{\sigma} = \frac{530-500}{20} = \frac{30}{20} = 1.5$

$P(x > 530) = P(z > 1.5)$

$= 1 - P(z \leq 1.5)$

$= 1 - 0.9332 = 0.668$

$= 6.7\%$

(ii) $\mu = 500g$, $\bar{x} = 495g, \sigma = 20g$ and $n = 40$

Sample statistic, $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{495-500}{\frac{20}{\sqrt{40}}} = \frac{-5}{3.1622} = -1.58$

(iii) $p - value = 2 \times P(z > |z_1|)$

$= 2 \times P(z > |-1.58|)$

$= 2 \times [1 - P(z \leq |1.58|)$

$= 2 \times [1 - 0.9429)$

$= 0.114$

(iv) Since $p > 0.05$ we accept the null hypothesis that $\mu = 500g$. The result is not significant, the mean weight has not changed.

5. (a) Mean = μ, the standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

(i) When n is large (n > 30) the distribution is Normal

(ii) When the population distribution is normal the distribution of the sample means is Normal. The Central Limit Theorem can be applied to (i) if n > 30. If the underlying population is normal the distribution of the sample means is always normal.

(b) (i) $\mu = €37, \sigma = €8.5$ *and* $n = 100$

Greater than €37.5 $\Rightarrow \bar{x} > €37.5$

Standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{37.5-37}{\frac{8.5}{\sqrt{100}}} = \frac{0.5}{0.85} = 0.5882$

$P(\bar{x} > 37.5) = P(z > 0.59)$

$= 1 - P(z \leq .59)$

$= 1 - 0.7224 = 0.278$

(ii) $P(\bar{x} > 37.5) < 0.06$

$P(z > z_1) = [1 - P(z \leq z_1)] = 0.06$

$P(z \leq z_1) = 0.94$

$z_1 = 1.55$

$z_1 = 1.55 = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{37.5-37}{\frac{8.5}{\sqrt{n}}}$

$1.55 \times \frac{8.5}{\sqrt{n}} = 0.5$

$\sqrt{n} = 26.35$

n = 694.3 = 695

6. (i) The lower quartile is €12.80 ⟹75% earn more than this amount

$P(x > 12.8) = 0.75$

(ii) 4 out of six earn more than €12.80 ⟹ $0.75 \times 0.75 \times 0.75 \times 0.75 \times 0.25 \times 0.25 \times$

$= 0.2966$

(iii) The distribution of the sample means will be normal with a mean of €22.05 and the standard

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10.64}{\sqrt{200}} = 0.7524$

(iv) $P(\bar{x} > €23)$

Standard unit $z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}} = \frac{23-22.05}{\frac{10.64}{\sqrt{200}}} = \frac{0.95}{0.7524} = 1.2626 = 1.26$

$P(\bar{x} > €23) = P(z > 1.26) = [1 - P(z \leq 1.26)]$

$= 1 - 0.8962$

$= 0.1038$

Number of samples = 0.1038(1000) = 103.8 = 104

7. $\mu = 3.05kg, \sigma = 0.08kg$

(i) $x = 3.11kg$

Standard unit $z = \frac{x-\mu}{\sigma} = \frac{3.11-3.05}{0.08} = \frac{0.06}{0.08} = 0.75$

$P(x < 3.11) = P(z \leq 0.75) = 0.7734 = 77.34\%$

(ii) $P(3.00 < x < 3.15)$

For $x = 3.00$, standard unit $z = \frac{x-\mu}{\sigma} = \frac{3.00-3.05}{0.08} = \frac{-0.05}{0.08} = -0.625$

For $x = 3.15$, standard unit $z = \frac{x-\mu}{\sigma} = \frac{3.15-3.05}{0.08} = \frac{0.1}{0.08} = 1.25$

$P(3.00 < x < 3.15) = P(-0.625 < z < 1.25)$

$= P(z \leq 1.25) - P(z < -0.625)$

$= P(z \leq 1.25) - P(z > 0.625)$

$= P(z \leq 1.25) - [1 - P(z \leq 0.625)]$

$= 0.8944 - [1 - 0.7357]$

$= 1.63$

8. (a) $\mu = 65min, \sigma = 60min$

(i) $x = 185$

Standard unit $z = \frac{x-\mu}{\sigma} = \frac{185-65}{60} = \frac{120}{60} = 2$

$P(x > 185) = P(z > 2) = [1 - P(z \leq 2)]$

$= 1 - 0.9772$

$= 0.0228$

(ii) $P(50 < x < 125)$

For $x = 50$, standard unit $z = \frac{x-\mu}{\sigma} = \frac{50-65}{60} = \frac{-15}{60} = -0.25$

For $x = 125$, standard unit $z = \frac{x-\mu}{\sigma} = \frac{125-65}{60} = \frac{60}{60} = 1$

$P(3.00 < x < 3.15) = P(-0.25 < z < 1)$

$= P(z \leq 1) - P(z < -0.25)$

$= P(z \leq 1) - P(z > 0.25)$

$= P(z \leq 1) - [1 - P(z \leq 0.25)]$

$= 0.8413 - [1 - 0.5987]$

$= 0.44$

(iii) $P(\bar{x} < 70)$ from a sample of 90.

Standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{70-65}{\frac{60}{\sqrt{90}}} = \frac{5}{6.3245} = 0.7906$

$P(\bar{x} < 70) = P(z \leq 0.7906) = 0.785$

(b) (i) The standard deviation is so big that there are only $\frac{65}{60} = 1.083$ standard deviations above zero.

$$P(z < -1.083) = P(z > 1.083) = [1 - P(z \leq 1.083)]$$
$$= 1 - 0.8599$$
$$= 0.14 \text{ at time} = 0 \text{ minutes}$$

This means that there is a probability of 0.14 of negative times, which are impossible.

(ii) A large sample of 90 $\Longrightarrow$ the mean is approximately normally distributed.

9. $\mu = 60g, \sigma = 15g$

(i) $x = 45g$

Standard unit $z = \frac{x-\mu}{\sigma} = \frac{45-60}{15} = \frac{-15}{15} = -1$

$P(x < 45g) = P(z \leq -1) = P(z > 1)$

$= [1 - P(z \leq 1)]$

$= 1 - 0.8413 = 0.1587$

(ii) $P(\bar{x} < 58g)$ from a sample of 50.

Standard unit $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{58-60}{\frac{15}{\sqrt{50}}} = \frac{-2}{2.1213} = -0.94$

$P(\bar{x} < 58) = P(z < -0.94) = P(z > 0.94)$

$= [1 - P(z \leq 0.94)]$

$= 1 - 0.8264 = 0.1736$

(iii) $P(x < 45g(small)) = 0.1587$

$\Longrightarrow P(x\ medium\ or\ large) = 1 - 0.1587 = 0.8413$

Equal probabilities $\Longrightarrow \frac{0.8413}{2} = 0.4206$ in each group

$\Rightarrow P(x \text{ small or medium}) = 0.1587 + 0.4206 = 0.5793$

$P(z) = 0.5793 \quad \Rightarrow z = 0.2$

Standard unit $z = \frac{x-\mu}{\sigma} = \frac{x-60}{15} = 0.2$

$\Rightarrow \quad x = 0.2(15) + 60 = 63g$