Revision Exercise (Core)

Q1 (i) $y = x^2 + \frac{1}{x}$

$y = x^2 + x^{-1}$

$\frac{dy}{dx} = 2x - x^{-2} = 2x - \frac{1}{x^2}$

(ii) $y = (2x+3)^3$

$dy/dx = 3(2x+3)^2(2) = 6(2x+3)^2$

(iii) $y = \sqrt{1+3x}$

$y = (1+3x)^{1/2}$

$dy/dx = \frac{1}{2}(1+3x)^{-1/2} = \frac{1}{2\sqrt{1+3x}}$

Q2 $y = x^2 + 3x - 4$

$f(x+h) = (x+h)^2 + 3(x+h) - 4$

$= x^2 + 2hx + h^2 + 3x + 3h - 4$

$f(x+h) - f(x) = (x^2 + 2hx + h^2 + 3x + 3h - 4) - (x^2 + 3x - 4)$

$= 2hx + h^2 + 3h$

$\frac{f(x+h) - f(x)}{h} = \frac{2hx + h^2 + 3h}{h} = \underline{2x + h + 3}$

$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2x + 3$

Q3 (i) $y = \frac{1}{3}(x+2)^3$

$dy/dx = 1(x+2)^2(1) = (x+2)^2$

(ii) $y = \frac{2x}{x+1}$

$$dy/dx = \frac{(x+1)(2) - (2x)(1)}{(x+1)^2}$$

$$= \frac{2x+2-2x}{(x+1)^2} = \frac{2}{(x+1)^2}$$

Q4 (i) $f(x) = 2x^2 - \frac{3}{x^2}$

$f(x) = 2x^2 - 3x^{-2}$

$f'(x) = 4x + 6x^{-3} = 4x + \frac{6}{x^3}$

(ii) $y = 4\sin 6x$

$dy/dx = 4\cos 6x(6) = 24\cos 6x$

(iii) $y = 3e^{x^2}$

$dy/dx = 3e^{x^2}(2x) = 6xe^{x^2}$

Q5 $y = \dfrac{2x+3}{x-4}$

$$\frac{dy}{dx} = \frac{(x-4)(2) - (2x+3)(1)}{(x-4)^2}$$

$$\frac{2x-8-2x-3}{(x-4)^2} = \frac{-11}{(x-4)^2}$$

$\Rightarrow k = -11$.

Q6 $y = 6x^2 - x^3$

Gradient is 12 $\Rightarrow dy/dx = 12$

$dy/dx = 12x - 3x^2 = 12$

$3x^2 - 12x + 12 = 0$

$(\div 3)$ $x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$.

find y: $y = 6x^2 - x^3$

$x = 2$ $y = 6(2)^2 - (2)^3 = 24 - 8 = 16$.

$P(2, 16)$ $m =$

Eqn $y - 16 = 12(x-2)$

$y - 16 = 12x - 24$

$12x - y - 8 = 0$.

Q7 (i) $y = 3x^2 - x + 3/x$

$y = 3x^2 - x + 3x^{-1}$

$dy/dx = 6x - 1 - 3x^{-2} = 6x - 1 - 3/x^2$

(ii) $y = \dfrac{3x^2}{x-1}$

$\dfrac{dy}{dx} = \dfrac{(x-1)(6x) - (3x^2)(1)}{(x-1)^2}$

$\dfrac{6x^2 - 6x - 3x^2}{(x-1)^2}$

$= \dfrac{3x^2 - 6x}{(x-1)^2}$

(iii) $y = \cos^2 4x$

$\dfrac{dy}{dx} = 2\cos 4x(-\sin 4x)(4)$

$= -8\cos 4x \sin 4x$

Q8 $\quad y = \dfrac{4x^2+6}{x}$

$y = \dfrac{4x^2}{x} + \dfrac{6}{x} = 4x + 6x^{-1}$

$dy/dx = 4 - 6x^{-2}$

$= 4 - 6/x^2$

Q9 $\quad f(x) = a \sin 3x$

$f'(x) = a \cos 3x(3)$

$= 3a \cos 3x$

$f'(\pi) = 2$

$3a \cos(3\pi) = 2$

$3a(-1) = 2$

$-3a = 2$

$a = -2/3$

Q10 $\quad y = x \sin 2x$

$\dfrac{dy}{dx} = x(\cos 2x)(2) + \sin 2x(1)$

$= 2x \cos 2x + \sin 2x$

at $x = \pi/3$ $\quad 2\left(\frac{\pi}{3}\right) \cos 2\left(\frac{\pi}{3}\right) + \sin 2\left(\frac{\pi}{3}\right)$

$\frac{2\pi}{3} \cos 2\pi/3 + \sin 2\pi/3$

$2\pi/3 \left(-\frac{1}{2}\right) + \frac{\sqrt{3}}{2} = \frac{-\pi}{3} + \frac{\sqrt{3}}{2}$

Q11 $\frac{dy}{dx} = (x+1)(x-2)$

$P(1,2)$ at $x=1$ slope $= (1+1)(1-2)$

$= 2(-1) = -2.$

Eqn: $y-2 = -2(x-1)$

$y-2 = -2x+2$

$2x+y-4=0.$

Q12 $f(x) = \sqrt{x} + \frac{1}{x^2}$

$f(x) = (x)^{\frac{1}{2}} + x^{-2}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-3}$

$= \frac{1}{2\sqrt{x}} - \frac{2}{x^3}$

~~at~~ $f'(4) = \frac{1}{2\sqrt{4}} - \frac{2}{4^3}$

$= \frac{1}{4} - \frac{2}{64} = \frac{16-2}{64} = \frac{14}{64} = 7/32$

Q13 $y = 2x^2 - 1$

(i) $x = 1$ $\quad y = 2(1)^2 - 1 = 1$ $\quad$ pt $(1, 1)$

$x = 4$ $\quad y = 2(4)^2 - 1 = 31$ $\quad$ pt $(4, 31)$

Average rate of change (slope) $= \frac{31-1}{4-1} = \frac{30}{3} = 10.$

(ii) $y = 2x^2 - 1$

$dy/dx = 4x$

at $x = 4$ $\quad dy/dx = 4(4) = 16.$

Q14 $y = \tan^{-1}(5x)$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ | $u = 5x$ $\quad \frac{du}{dx} = 5$; $y = \tan^{-1} u$ $\quad \frac{dy}{du} = \frac{1}{1+u^2}$

$\frac{dy}{dx} = \frac{1}{1+u^2} \cdot 5$

$= \frac{1}{1+25x^2} \cdot 5 = \frac{5}{1+25x^2}$

Q15 $y = 2x^2 - 2x + 3$ $P(1,3)$

$dy/dx = 4x - 2$

at $x = 1$ $m = 4(1) - 2 = 2$

eqn : $y - 3 = 2(x - 1)$

$y - 3 = 2x - 2$

$2x - y + 1 = 0$

Q16 $f(x) = 2x^{-3} + \frac{k}{2}x^{-2} - x$

$f'(x) = -6x^{-4} + \frac{-\cancel{2}k}{\cancel{2}}x^{-3} - 1$

$= \frac{-6}{x^4} - \frac{k}{x^3} - 1$

$f'(-2) = 0$: $\frac{-6}{(-2)^4} - \frac{k}{(-2)^3} - 1 = 0$

$\frac{-6}{16} - \frac{k}{-8} - 1 = 0$

$\frac{-3}{8} + \frac{k}{8} - 1 = 0$

$\frac{-3 + k - 8}{8} = 0$

$-3 + k - 8 = 0$

$-11 + k = 0$

$k = 11$