

Ex 2.4

Q1 (i) $y = (1-x)(2-x^2)$

$$\begin{aligned} \frac{dy}{dx} &= (1-x)(-2x) + (2-x^2)(-1) \\ &= -2x + 2x^2 - 2 + x^2 \\ &= 3x^2 - 2x - 2 \end{aligned}$$

(ii) $y = (x^3-1)(2x+1)$

$$\begin{aligned} \frac{dy}{dx} &= (x^3-1)(2) + (2x+1)(3x^2) \\ &= 2x^3 - 2 + 6x^3 + 3x^2 \\ &= 8x^3 + 3x^2 - 2 \end{aligned}$$

Q2 (i) $f(x) = \frac{2x^3}{1-2x}$

$$\begin{aligned} f'(x) &= \frac{(1-2x)(6x^2) - (2x^3)(-2)}{(1-2x)^2} \\ &= \frac{6x^2 - 12x^3 + 4x^3}{(1-2x)^2} \\ &= \frac{-8x^3 + 6x^2}{(1-2x)^2} \end{aligned}$$

$$(vi) f(x) = \frac{3x+2}{x^2-3}$$

$$f'(x) = \frac{(x^2-3)(3) - (3x+2)(2x)}{(x^2-3)^2}$$

$$= \frac{3x^2-9-6x^2-4x}{(x^2-3)^2}$$

$$= \frac{-3x^2-4x-9}{(x^2-3)^2}$$

$$\textcircled{04} \quad y = \sqrt{x}(2x-1)$$

$$= (x^{\frac{1}{2}})(2x-1)$$

$$\frac{dy}{dx} = (x^{\frac{1}{2}})(2) + (2x-1)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= 2\sqrt{x} + (2x-1)\left(\frac{1}{2\sqrt{x}}\right)$$

$$= 2\sqrt{x} + \frac{2x}{\sqrt{x}} - \frac{1}{2\sqrt{x}}$$

$$\frac{4x + 2x - 1}{2\sqrt{x}}$$

$$= \frac{6x-1}{2\sqrt{x}}$$

Q6 $y = \frac{x}{1-x^2}$ show $\frac{dy}{dx} > 0$

$$\frac{dy}{dx} = \frac{(1-x^2)(1) - (x)(-2x)}{(1-x^2)^2}$$

$$= \frac{1-x^2+2x^2}{(1-x^2)^2}$$

$$\frac{1+x^2}{(1-x^2)^2} > 0 \text{ as } (\text{real})^2 > 0$$

Q7 (iv) $y = (x^2-1)^2$

$$\frac{dy}{dx} = 2(x^2-1)(2x)$$

$$= (4x)(x^2-1) = 4x^3 - 4x$$

(v) $y = (2x^2+3)^4$

$$\frac{dy}{dx} = 4(2x^2+3)^3 (4x)$$

$$= (16x)(2x^2+3)^3$$

(vi) $y = (1-3x)^5$

$$\frac{dy}{dx} = 5(1-3x)^4 (-3)$$

$$= (-15)(1-3x)^4$$

Q8 (iii) $f(x) = \sqrt{x^3 - 2x}$
 $= (x^3 - 2x)^{1/2}$

$$f'(x) = \frac{1}{2} (x^3 - 2x)^{-1/2} (3x^2 - 2)$$

$$= \frac{3x^2 - 2}{2\sqrt{x^3 - 2x}}$$

Q9
(ii)

$$y = (x^2 - 1)(3x + 2)^2$$

$$\frac{dy}{dx} = (x^2 - 1)2(3x + 2)(3) + (3x + 2)^2(2x)$$

$$= 6(x^2 - 1)(3x + 2) + (2x)(3x + 2)^2$$

Q11

$$y = \frac{(2x - 1)^2}{3x + 4} \quad \frac{dy}{dx} \text{ at } x = 0$$

$$\frac{dy}{dx} = \frac{(3x + 4)2(2x - 1)(2) - (2x - 1)^2(3)}{(3x + 4)^2}$$

$$= \frac{4(3x + 4)(2x - 1) - 3(2x - 1)^2}{(3x + 4)^2}$$

at $x = 0$ $\frac{4(4)(-1) - 3(-1)^2}{(4)^2}$

$$f'(0) = \frac{-16 - 3}{16} = \frac{-19}{16}$$

Q13 $f(x) = \frac{x\sqrt{x+1}}{1-x}$

$$f'(x) = x \cdot \frac{1}{2}(x+1)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}}(1)$$

$$= \frac{x}{2\sqrt{x+1}} + \frac{\sqrt{x+1}}{1}$$

$$= \frac{x + (2\sqrt{x+1})(\sqrt{x+1})}{2\sqrt{x+1}}$$

$$= \frac{x + 2(x+1)}{2\sqrt{x+1}}$$

$$= \frac{x + 2x + 2}{2\sqrt{x+1}}$$

$$= \frac{3x + 2}{2\sqrt{x+1}}$$

Q14

$$4x^2 + 2xy = 5$$

$$2xy = 5 - 4x^2$$

$$y = \frac{5 - 4x^2}{2x}$$

$$\frac{dy}{dx} = \frac{(2x)(-8x) - (5 - 4x^2)(2)}{(2x)^2}$$

$$= \frac{-16x^2 - 10 + 8x^2}{4x^2}$$

$$= \frac{-8x^2 - 10}{4x^2}$$

$$= \frac{-4x^2 - 5}{2x^2}$$

Q18 $y = (x-1)^{3/2} - 3(x-1)^{1/2}$

$$\frac{dy}{dx} = \frac{3}{2}(x-1)^{1/2} - \frac{3}{2}(x-1)^{-1/2}$$

$$= \frac{3\sqrt{x-1}}{2} - \frac{3}{2\sqrt{x-1}}$$

$$= \frac{3(\sqrt{x-1})(\sqrt{x-1}) - 3}{2\sqrt{x-1}}$$

$$= \frac{3(x-1) - 3}{2\sqrt{x-1}}$$

$$= \frac{3x - 3 - 3}{2\sqrt{x-1}} = \frac{3x - 6}{2\sqrt{x-1}} = \frac{3(x-2)}{2\sqrt{x-1}} \quad \text{QED}$$

Q20

$$f(x) = \sqrt{\frac{4x}{x+3}} \quad f'(1)$$

$$f(x) = \left(\frac{4x}{x+3}\right)^{1/2}$$

$$f'(x) = \frac{1}{2} \left(\frac{4x}{x+3}\right)^{-1/2} \left(\frac{(x+3)(4) - (4x)(1)}{(x+3)^2}\right)$$

$$= \frac{1}{2} \left(\frac{4x}{x+3}\right)^{-1/2} \left(\frac{4x+12-4x}{(x+3)^2}\right)$$

$$= \frac{1}{2} \left(\frac{4x}{x+3}\right)^{-1/2} \left(\frac{12}{(x+3)^2}\right)$$

$$f'(1) = \frac{1}{2} \left(\frac{4}{4}\right)^{-1/2} \left(\frac{12}{4^2}\right)$$

$$= \frac{1}{2} (1) \left(\frac{12}{16}\right)$$

$$= \left(\frac{1}{2}\right) \left(\frac{3}{4}\right) = \frac{3}{8}$$

Q 22

(a) $f(x) = 3x+1$ $g(x) = x^2-2$

$$f(g(x)) = 3(x^2-2)+1$$

$$= 3x^2-6+1$$

$$= 3x^2-5$$

$$p(x) = 3x^2-5$$

$$p'(x) = 6x$$

$$g(f(x)) = (3x+1)^2-2$$

$$9x^2+6x+1-2$$

$$9x^2+6x-1$$

$$q(x) = 9x^2+6x-1$$

$$q'(x) = 18x+6$$

(b)

Solve $p'(x) = q'(x)$

$$6x = 18x + 6$$

$$-12x = 6$$

$$x = \frac{-6}{12}$$

$$x = -\frac{1}{2}$$