Ex 2·6

Q1 (vii) $y = \sin \frac{1}{2}x$

$\frac{dy}{dx} = \cos \frac{1}{2}x \cdot \frac{1}{2} = \frac{\cos \frac{1}{2}x}{2}$

(viii) $y = \cos(x^2-1)$

$\frac{dy}{dx} = -\sin(x^2-1)(2x) = -2x\sin(x^2-1)$

(ix) $y = \sin 2x + \cos 4x$

$\frac{dy}{dx} = \cos(2x)(2) + [-\sin(4x)(4)]$

$= 2\cos 2x - 4\sin 4x$

Q2 (i) $y = \sin^2 x$

$\frac{dy}{dx} = 2\sin x(\cos x)$

(v) $y = \cos^2(2x+1)$

$\frac{dy}{dx} = 2\cos(2x+1)(-\sin(2x+1))(2)$

$= -4\cos(2x+1)\sin(2x+1)$

Q3 (iii) $y = \cos 4\theta - \cos \frac{\theta}{4}$

$\frac{dy}{dx} = (-\sin 4\theta)(4) - (-\sin \frac{\theta}{4})(\frac{1}{4})$

$= -4 \sin 4\theta + \frac{1}{4} \sin \frac{\theta}{4}$

(iv) $y = \tan^3\theta + 5$

$\frac{dy}{dx} = 3 \tan^2\theta (\sec^2\theta)$

Q4

(ii) $y = x^2 \cos x$

$\frac{dy}{dx} = (x^2)(-\sin x) + (\cos x)(2x)$

$= -x^2 \sin x + 2x \cos x$

(iii) $y = (x+3)(\sin x)$

$\frac{dy}{dx} = (x+3)(\cos x) + (\sin x)(1)$

$= (x+3) \cos x + \sin x$.

Q6 $y = \cos x \tan x$

$= \cancel{\cos x} \cdot \dfrac{\sin x}{\cancel{\cos x}}$

$y = \sin x$

$\dfrac{dy}{dx} = \cos x$

$\dfrac{dy}{dx} = \cos x \sec^2 x + \tan x (-\sin x)$

$= \cos x \sec^2 x - \tan x \sin x$

$= \cos x \dfrac{1}{\cos^2 x} - \dfrac{\sin x}{\cos x} \sin x$

$\dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$

$\dfrac{1 - \sin^2 x}{\cos x} = \dfrac{\cos^2 x}{\cos x} = \cos x$

Q7 (i) $y = \sin 2x$

$\dfrac{dy}{dx} = \cos(2x)(2) = 2\cos 2x$

@ $x = \pi$ $\quad 2\cos(2\pi) = 2 \times 1 = 2$

Q7 (ii) $y = x \cos x$

$\dfrac{dy}{dx} = x(-\sin x) + \cos x (1)$

$= -x \sin x + \cos x$

at $x = \pi \Rightarrow -\pi \sin \pi + \cos \pi$

$-\pi(0) + (-1) = -1$

(iii) $y = \sin^2 x$

$dy/dx = 2 \sin x (\cos x)$

at $x = \pi \Rightarrow 2(\sin \pi)(\cos \pi)$

$(2)(0)(-1) = 0$

Q8 given $\tan x = \frac{\sin x}{\cos x}$ show $\frac{d(\tan x)}{dx} = \sec^2 x$

$$y = \tan x$$

$$y = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= 1 + \frac{\sin^2 x}{\cos^2 x}$$

$$= 1 + \tan^2 x$$

$$= \sec^2 x \quad \text{(Tables pg 13)} \quad \text{QED}$$

Q9 $f(x) = (\sin x + 1)^2$

$$f'(x) = 2(\sin x + 1)(\cos \theta)$$

$$= 2\cos x(\sin x + 1)$$

@ $x = \pi/6$ $\Rightarrow 2\cos \pi/6 (\sin \pi/6 + 1)$

$$2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2} + 1\right)$$

$$\sqrt{3}\left(\frac{3}{2}\right) = \frac{3\sqrt{3}}{2}$$

Q10 $y = \sin x + 3\cos x$

$\frac{dy}{dx} = +\cos x - 3\sin x$

Show $\cos x \frac{dy}{dx} + y \sin x = 1$

$(\cos x)(\cos x - 3\sin x) + (\sin x)(\sin x + 3\cos x)$

$= \cos^2 x - \cancel{3\sin x\cos x} + \sin^2 x + \cancel{3\sin x\cos x}$

$= \cos^2 x + \sin^2 x = 1$. QED.

Q11 $y = \sin 2x - 2x$

$\frac{dy}{dx} = \cos 2x(2) - 2$

$= 2\cos 2x - 2$

$= 2(1 - 2\sin^2 A) - 2.$

$= \cancel{2} - 4\sin^2 A - \cancel{2}$

$= -4\sin^2 A$

Q12 $y = \cos\left(\frac{1}{4}\pi x\right)$

$$\frac{dy}{dx} = -\sin\left(\frac{1}{4}\pi x\right)\left(\frac{1}{4}\pi\right)$$

at $x = 4$

$$\frac{dy}{dx} = -\sin\left(\frac{1}{4}\quad\right)\left(\frac{\pi}{4}\right)$$

$$= -\sin\pi\left(\frac{\pi}{4}\right)$$

$$= -(0)\frac{\pi}{4} = 0$$

Q13 $f(x) = \cos^3 2x$

$$f'(x) = 3\cos^2 2x(-\sin 2x)(2)$$

$$= -6\cos^2 2x \sin 2x$$

at $x = \pi/6$ $= -6\cos^2\left(2\left(\frac{\pi}{6}\right)\right)\sin\left(2\left(\frac{\pi}{6}\right)\right)$

$$= -6\cos^2 \pi/3 \sin \pi/3$$

$$= -6\left(\frac{1}{2}\right)^2\left(\frac{\sqrt{3}}{2}\right)$$

$$= -6 \cdot \frac{1}{4} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{-3\sqrt{3}}{4}.$$

Q14 $f(x) = \cos 2x$ $g(x) = 2\sin^2 x$

(i) $f'(x) = -\sin 2x(2)$ $g'(x) = 4\sin x(\cos x)$

$= -2\sin 2x$

Show $f'(x) + g'(x) = 0$

$-2\sin 2x + 4\sin x\cos x = 0$

$-2(2\sin x\cos x) + 4\sin x\cos x = 0$

$-4\sin x\cos x + 4\sin x\cos x = 0$

$0 = 0$ QED.

Q15 $y = \sin 3x$

$dy/dx = 3\cos 3x$

$\frac{d^2y}{dx^2} = 3(-\sin(3x)(3)) = 3(-3\sin 3x) = -9\sin 3x$

$= -9y$.

Q16 $y = \tan x + \frac{1}{3}\tan^3 x$

$\frac{dy}{dx} = \sec^2 x + 3(\frac{1}{3})\tan^2 x(\sec^2 x)$

$= \sec^2 x + \tan^2 x\sec^2 x$

$= \sec^2 x(1 + \tan^2 x)$

$= \sec^2 x(\sec^2 x)$

$= \sec^4 x$.

Q17 $y = 3\sin x + k\sin 3x$

$\frac{dy}{dx} = 0$ when $x = \pi/3$

$\frac{dy}{dx} = 3\cos x + k\cos 3x(3)$

$= 3\cos x + 3k\cos 3x$

at $x = \pi/3$ $\quad 3\cos(\pi/3) + 3k\cos(3.\pi/3) = 0$

$3\cos\pi/3 + 3k\cos\pi = 0$

$3\left(\frac{1}{2}\right) + 3k(-1) = 0$

$3/2 - 3k = 0$

$\frac{3}{2} = 3k$

$3/6 = k$

$\frac{1}{2} = k.$