Ex 2.7

Q1 (i) $y = \sin^{-1}\frac{6x}{1}$ $\left(\frac{x}{a}\right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1^2-(6x)^2}} \cdot 6 = \frac{6}{\sqrt{1-36x^2}}$$

(ii) $y = \tan^{-1}\frac{3x}{1}$

$$\frac{dy}{dx} = \frac{1}{(1)^2+(3x)^2} \cdot 3 = \frac{3}{1+9x^2}$$

(iii) $y = \sin^{-1}\frac{(2x+1)}{1}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1^2-(2x+1)^2}} \cdot 2x = \frac{2x}{\sqrt{1-4x^2-4x-1}}$$

$$= \frac{2}{\sqrt{-4x^2-4x}}$$

(iv) $y = \tan^{-1}\frac{(x^2)}{1}$

$$\frac{dy}{dx} = \frac{1}{(1)^2+(x^2)^2} \cdot 2x = \frac{2x}{1+x^4}$$

Q4

(i) $f(x) = \sin^{-1}\frac{3/x}{1}$

$\frac{3}{x} = 3x^{-1}$

$\frac{dy}{dx} = -3x^{-2}$

$= \frac{-3}{x^2}$

$$f'(x) = \frac{1}{\sqrt{1^2-(3/x)^2}} \cdot \frac{-3}{x^2}$$

$$= \frac{1}{\sqrt{1-9/x^2}} \cdot \frac{-3}{x^2}$$

$$= \frac{1}{\sqrt{\frac{x^2-9}{x^2}}} \cdot \frac{-3}{x^2}$$

$$= \frac{1}{\frac{\sqrt{x^2-9}}{x}} \cdot \frac{-3}{x^2}$$

$$= \frac{x}{\sqrt{x^2-9}} \cdot \frac{-3}{x^2}$$

$$= \frac{-3}{x\sqrt{x^2-9}}$$

(ii)

$$f(x) = \tan^{-1}\frac{x/4}{1}$$

$y = \frac{1}{4}x \quad \frac{dy}{dx} = \frac{1}{4}.$

$$f'(x) = \frac{1}{1^2+\left(\frac{x}{4}\right)^2} \cdot \frac{1}{4}$$

$$= \frac{1}{1+\frac{x^2}{16}} \cdot \frac{1}{4}$$

$$= \frac{1}{\frac{16+x^2}{16}} \cdot \frac{1}{4}$$

$$= \frac{16^4}{16+x^2} \cdot \frac{1}{4}$$

$$= \frac{4}{16+x^2}$$

Q6 $y = (\sin^{-1} x)^2$ Show $\frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = 2(\sin^{-1}x)^1 \cdot \frac{1}{\sqrt{1-x^2}} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}}$$

Q8 $f(x) = \tan^{-1}(\cos x)$ $f'(\frac{\pi}{6})$

$$f'(x) = \frac{1}{1+\cos^2 x} \cdot -\sin x$$

$$= \frac{-\sin x}{1+\cos^2 x}$$

$$f'(\tfrac{\pi}{6}) = \frac{-\sin(\frac{\pi}{6})}{1+\cos^2(\frac{\pi}{6})} = \frac{-\frac{1}{2}}{1+(\frac{\sqrt{3}}{2})^2}$$

$$= \frac{-\frac{1}{2}}{1+\frac{3}{4}} = \frac{-\frac{1}{2}}{\frac{7}{4}} = -\frac{1}{2}\times\frac{4}{7} = \frac{-2}{7}$$

Q10 $y = \tan^{-1}(3x^2)$ find dy/dx @ $x = 1/3$

$$\frac{dy}{dx} = \frac{1}{1^2+(3x^2)^2} \cdot 6x = \frac{6x}{1+9x^4}$$

@ $x = \frac{1}{3}$ $= \frac{6(\frac{1}{3})}{1+9(\frac{1}{3})^4} = \frac{2}{1+9(\frac{1}{81})} = \frac{2}{1+\frac{1}{9}}$

$$= \frac{2}{\frac{10}{9}} = 2\times\frac{9}{10} = \frac{9}{5}$$