Ex 2.8

Q1 (iv) $y = e^{2x+4}$

$\frac{dy}{dx} = e^{2x+4}(2) = 2e^{2x+4}$

(v) $y = e^{x^2+3x}$

$\frac{dy}{dx} = e^{x^2+3x}(2x+3) = (2x+3)e^{x^2+3x}$

(vi) $y = e^{\sin x}$

$\frac{dy}{dx} = e^{\sin x}(\cos x) = (\cos x)e^{\sin x}$

Q2 (i) $y = e^{x/2}$

$\frac{dy}{dx} = e^{x/2}\left(\frac{1}{2}\right) = \frac{1}{2}e^{x/2}$

(ii) $y = e^{\sin^2 x}$

$\frac{dy}{dx} = e^{\sin^2 x}(2\sin x.\cos x)$

$= 2\sin x \cos x \left(e^{\sin^2 x}\right)$

(iii) $y = xe^{2x}$

$dy/dx = x(e^{2x})(2) + e^{2x}(1)$

$= 2xe^{2x} + e^{2x}$

$= e^{2x}(2x+1)$

Q3 (i) $y = e^{2x}\sin x$

$$\frac{dy}{dx} = (e^{2x})(\cos x) + (\sin x)(e^{2x})(2)$$

$$= e^{2x}\cos x + 2e^{2x}\sin x$$

$$= e^{2x}(\cos x + 2\sin x)$$

(ii) $y = (e^x - 1)^2$

$$\frac{dy}{dx} = 2(e^x - 1)\,e^x = 2e^x(e^x - 1)$$

(iii) $y = \dfrac{e^{2x+1}}{e^x}$

$$\frac{dy}{dx} = \frac{(e^x)(e^{2x+1})(2) - (e^{2x+1})(e^x)}{(e^x)^2}$$

$$= \frac{2 . e^x . e^{2x+1} - e^{2x+1}e^x}{(e^x)^2}$$

$$= \frac{(e^{2x+1})(e^x)(2-1)}{(e^x)^2}$$

$$= \frac{e^{2x+1}\cancel{e^x}}{e^x\cancel{e^x}} = \frac{e^{2x+1}}{e^x} = e^{x+1}$$

$\boxed{\text{OR}}$ $y = \dfrac{e^{2x+1}}{e^x} = e^{x+1}$

$$\frac{dy}{dx} = e^{x+1}(1) = e^{x+1}$$

Q4 (i) $y = e^{2x}(1+e^{x}) = e^{2x} + e^{3x}$

$$dy/dx = e^{2x}(2) + e^{3x}(3)$$
$$= 2e^{2x} + 3e^{3x}$$

(ii) $t = \dfrac{e^{2x}}{x}$

$$\frac{dt}{dx} = \frac{x(e^{2x})(2) - e^{2x}(1)}{x^2}$$

$$= \frac{2x.e^{2x} - e^{2x}}{x^2}$$

$$= \frac{e^{2x}(2x-1)}{x^2}$$

(iii) $y = x^2 e^{\cos x}$

$$dy/dx = x^2(e^{\cos x})(-\sin x) + e^{\cos x}(2x)$$
$$= e^{\cos x}(2x - x^2 \sin x)$$
$$x e^{\cos x}(2 - x \sin x)$$

Q5 $y = e^{3x} \sin(\pi x)$ $\qquad \frac{dy}{dx}$ at $x = 1$

$$\frac{dy}{dx} = e^{3x} (\cos \pi x) \pi + (\sin \pi x)(e^{3x})(3)$$

at $x = 1$ $\qquad e^{3} (\cos \pi)(\pi) + (\sin \pi)(e^{3})(3)$

$$= e^{3} (-1) \pi + (0) e^{3} (3)$$

$$= -\pi e^{3}$$

Q6 $y = e^{2x}$ $\qquad \frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = e^{2x}(2) = 2e^{2x}$$

$$\frac{d^2y}{dx^2} = 2e^{2x}(2) = 4e^{2x}$$

Show $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

$$4e^{2x} - 3(2e^{2x}) + 2(e^{2x}) = 0$$

$$4e^{2x} - 6e^{2x} + 2e^{2x} = 0$$

$$-2e^{2x} + 2e^{2x} = 0$$

$$0 = 0 \quad \text{QED}$$

Q7 $y = e^x(\cos x - \sin x)$

$$\frac{dy}{dx} = e^x(-\sin x - \cos x) + (\cos x - \sin x)e^x$$

$$= e^x(-\sin x \cancel{- \cos x} + \cancel{\cos x} - \sin x)$$

$$= e^x(-2\sin x)$$

$$= -2e^x \sin x \qquad \text{QED}$$

Q8 $y = xe^x$ Show $\frac{d^2y}{dx^2} + y = 2\,dy/dx$

$$\frac{dy}{dx} = xe^x + e^x = e^x(x+1)$$

$$\frac{d^2y}{dx^2} = e^x(1) + (x+1)e^x$$

$$= e^x(1+x+1) = e^x(x+2)$$

$$\frac{d^2y}{dx^2} + y = 2\frac{dy}{dx}$$

$$e^x(x+2) + xe^x = 2(e^x)(x+1)$$

$$e^x(x+2+x) = e^x(2x+2)$$

$$e^x(2x+2) = e^x(2x+2)$$

QED

Q9 $f(x) = e^{2x} - ae^{x}$ Show $f'(x) = 0$ when $e^{x} = \frac{a}{2}$

$$f'(x) = e^{2x}(2) - ae^{x}$$
$$= 2e^{2x} - ae^{x}$$

when $e^{x} = \frac{a}{2}$

$$f'(x) = 2\left(\frac{a}{2}\right)^2 - a\left(\frac{a}{2}\right)$$
$$= \frac{2a^2}{4} - \frac{a^2}{2}$$
$$= \frac{2a^2 - 2a^2}{4} = \frac{0}{4} = 0$$

Q10 $y = e^{mx}$

$$\frac{dy}{dx} = e^{mx}(m) = me^{mx}$$
$$\frac{d^2y}{dx^2} = me^{mx}(m) = m^2e^{mx}$$

find m if $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$

$$m^2e^{mx} - 3me^{mx} - 4e^{mx} = 0$$
$(\div e^{mx})$
$$m^2 - 3m - 4 = 0$$
$$(m - 4)(m + 1) = 0$$
$$m = 4 \qquad m = -1$$

Q11 $f(x) = \dfrac{e^{x}+e^{-x}}{2} = \frac{1}{2}(e^{x}+e^{-x})$

$f'(x) = \frac{1}{2}(e^{x}.1+e^{-x}.(-1))$

$= \frac{1}{2}(e^{x}-e^{-x})$

$f''(x) = \frac{1}{2}(e^{x}.(1)-e^{-x}.(-1))$

$= \frac{1}{2}(e^{x}+e^{-x}) = f(x)$

QED

OR Using Quotient Rule.

$f(x) = \dfrac{e^{x}+e^{-x}}{2}$

$f'(x) = \dfrac{2(e^{x}(1)+e^{-x}(-1)) - \overbrace{(e^{x}+e^{-x})(0)}^{0}}{2^2}$

$\dfrac{2(e^{x}-e^{-x})}{4} = \dfrac{e^{x}-e^{-x}}{2}$

$f''(x) = \dfrac{2(e^{x}(1)-e^{-x}(-1)) - (e^{x}-e^{-x})(0)}{2^2}$

$= \dfrac{2(e^{x}+e^{-x})}{4}$

$= \dfrac{e^{x}+e^{-x}}{2} = f(x)$ QED

Q12 $y = 3e^x - \sin x + 5$.

Tangent: $y - y_1 = m(x - x_1)$ require pt and m.

find m: $\frac{dy}{dx} = 3e^x - \cos x$

at $x = 0$ $m = 3e^0 - \cos 0$

$= 3 - 1$

$\boxed{m = 2}$

Find Point: $y = 3e^x - \sin x + 5$

at $x = 0$ $y = 3e^0 - \sin 0 + 5$

$= 3 - 0 + 5$

$= 8$

Point is $(0, 8)$

Eqn of Tangent: $y - 8 = 2(x - 0)$

$2x - y + 8 = 0$.

Q13 L_1: $y = 2e^x - x$ at Pt $(0, 2)$

$\frac{dy}{dx} = 2e^x - 1$ at $x = 0$ Slope $= 2 - 1 = \underline{\underline{1}}$

Eqn of Tangent: $y - 2 = 1(x - 0)$ $\boxed{y = x + 2}$

L_2: $y = \sin 2x - x^2$ at $(0, 0)$

$\frac{dy}{dx} = 2\cos 2x - 2x$ at $x = 0$, Slope $= 2\cos 0 - 0$

$= 2(1) = 2$ //

Eqn of Tangent: $y - 0 = 2(x - 0)$ $\boxed{y = 2x}$

Pt of Intersection

$y = x + 2$

$-y = 2x$

$0 = -x + 2$

$x = 2$.

find y.

$y = 2(2)$

$y = 4$

Pt of $\cap$ is $(2, 4)$

Q.E.D.