Ex 2.9

Q1 $y = \log_e 5x$

$\frac{dy}{dx} = \frac{1}{5x} \cdot 5 = \frac{1}{x}$

Q2 $y = \log_e(2x+3)$

$dy/dx = \frac{1}{2x+3} \cdot 2 = \frac{2}{2x+3}$

Q3 $y = \log_e(3x^2)$

$dy/dx = \frac{1}{3x^2} \cdot 6x = \frac{2}{x}$

Q4 $y = \log_e(\sin x)$

$dy/dx = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \text{Cotan}\, x$

Q5 $y = \log_e(x^2 - 6x)$

$dy/dx = \frac{1}{x^2-6x} \cdot 2x-6 = \frac{2x-6}{x^2-6x} = \frac{2(x-3)}{x(x-6)}$

Q6 $y = \log_e(\cos 3x)$

$dy/dx = \frac{1}{\cos 3x} \cdot -\sin 3x(3) = -3\frac{\sin 3x}{\cos 3x} = -3\tan 3x$

Q7 $y = x \log_e x$

$dy/dx = x\frac{1}{x} + \log_e x(1)$

$= 1 + \log_e x$

Q8 $y = x^2 \ln(3x)$

$$\frac{dy}{dx} = x^2 \frac{1}{3x}(3) + \ln(3x)2x$$

$$= x + 2x\ln 3x$$

Q9 $y = \frac{\ln x}{x}$

$$\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \ln x(1)}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

Q10 (i) $y = \log_e(3x+1)^3$

$$y = 3\log_e(3x+1)$$

$$\frac{dy}{dx} = 3 \cdot \frac{1}{3x+1} \cdot 3 = \frac{9}{3x+1}$$

(ii) $y = \log_e\left(\frac{2x+1}{1-3x}\right)$

$$y = \log_e(2x+1) - \log_e(1-3x)$$

$$\frac{dy}{dx} = \frac{1}{2x+1} \cdot 2 - \frac{1}{1-3x} \cdot (-3)$$

$$= \frac{2}{2x+1} + \frac{3}{1-3x}$$

$$\frac{2-6x+6x+3}{(2x+1)(1-3x)} = \frac{5}{(2x+1)(1-3x)}$$

(iii) $y = \log_e \sqrt{1+x^2}$

$y = \log_e (1+x^2)^{\frac{1}{2}}$

$y = \frac{1}{2} \log_e (1+x^2)$

$dy/dx = \frac{1}{2} \cdot \frac{1}{1+x^2} \cdot 2x$

$= \frac{\cancel{2}x}{\cancel{2}(1+x^2)} = \frac{x}{1+x^2}$

(iv) $y = \log_e \sqrt{\sin x}$

$y = \frac{1}{2} \log_e (\sin x)$

$dy/dx = \frac{1}{2} \cdot \frac{1}{\sin x} \cdot \cos x$

$= \frac{1}{2} \cdot \frac{\cos x}{\sin x}$

$= \frac{1}{2} \operatorname{Cotan} x$

(v) $y = \log_e (x^2+4)^2$

$y = 2 \log_e (x^2+4)$

$dy/dx = 2 \cdot \frac{1}{x^2+4} \cdot 2x$

$= \frac{4x}{x^2+4}$

(xi) $y = \log_e \sqrt{\frac{x}{1+x}}$

$y = \frac{1}{2} \log_e \left(\frac{x}{1+x}\right)$

$y = \frac{1}{2}\left[\log_e x - \log_e (1+x)\right]$

$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x} - \frac{1}{1+x}\right]$

$= \frac{1}{2}\left[\frac{1+x-x}{(x)(1+x)}\right]$

$= \frac{1}{2}\left[\frac{1}{(x)(1+x)}\right]$

$= \frac{1}{2x+2x^2}.$

Q11 $y = \ln 3x^4$

$\frac{dy}{dx} = \frac{1}{3x^4} \cdot 12x^3$

$= \frac{4}{x}$

$= 4x^{-1}$

$\frac{d^2y}{dx^2} = -4x^{-2}$

$= \frac{-4}{x^2}$

Q12 $y = \left[\log_e (x+4)\right]^2$

$$\frac{dy}{dx} = 2\left[\log_e(x+4)\right] \cdot \frac{1}{x+4} \cdot (1)$$

$$= \frac{2\log_e(x+4)}{x+4}$$

Q13 $y = x \log_e x$

$$\frac{dy}{dx} = x\frac{1}{x} + \log_e x(1)$$

$$= 1 + \log_e x$$

$$\frac{d^2y}{dx^2} = \frac{1}{x}$$

Q14 slope $= dy/dx$

$$y = \log_e x - 2x + x^2$$

$$dy/dx = \frac{1}{x} - 2 + 2x$$

at $x = 2$. $\frac{1}{2} - 2 + 4$

$= 2\frac{1}{2}$

Q15 $y = (\ln x)^2$

$\frac{dy}{dx} = 2(\ln x) \cdot \frac{1}{x}$

at $x = e$ $\Rightarrow 2\ln e \cdot \frac{1}{e}$

$= 2(1) \cdot \frac{1}{e} = \frac{2}{e}$

Q16 $y = \ln(1 + \sin t)$

$\frac{dy}{dt} = \frac{1}{1+\sin t} \cdot \cos t$

$= \frac{\cos t}{1 + \sin t}$

$\frac{d^2y}{dx^2} = \frac{(1+\sin t)(-\sin t) - (\cos t)(\cos t)}{(1+\sin t)^2}$

$= \frac{-\sin t - \sin^2 t - \cos^2 t}{(1+\sin t)^2}$

$= \frac{-1(\sin t + \underbrace{\sin^2 t + \cos^2 t}_{1})}{(1+\sin t)^2}$

$= \frac{-1(\sin t + 1)}{(1+\sin t)^2}$

$= \frac{-1}{1+\sin t}$

If $(1+\sin t)\frac{d^2y}{dx^2} + k = 0$

$\cancel{(1+\sin t)}\left(\frac{-1}{\cancel{1+\sin t}}\right) + k = 0$

$-1 + k = 0$

$k = 1$

Q17 $y = \ln(e^x \cos x)$

$y = \ln e^x + \ln \cos x$

$y = x + \ln \cos x$

$\frac{dy}{dx} = 1 + \frac{1}{\cos x} \cdot -\sin x$

$= 1 - \frac{\sin x}{\cos x}$

$= 1 - \tan x$