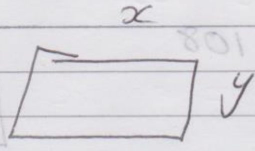
Q3



$P = 100$

$$2x + 2y = 100$$
$$2y = 100 - 2x$$
$$y = (50 - x)$$

$$\text{Area} = L \times W$$
$$A = (\cancel{x})(50 - x)$$
$$A = 50x - x^2$$

Max Area $\Rightarrow \frac{dA}{dx} = 0$

$$\frac{dA}{dx} = 50 - 2x = 0$$
$$50 = 2x$$
$$\underline{25 = x}$$

Show Max

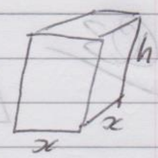
$$\frac{d^2A}{dx^2} = -2 < 0 \Rightarrow \text{Max.}$$

$$\Rightarrow \text{Area} = (x)(50 - x)$$
$$= (25)(25)$$
$$= 625\ m^2$$

Ex 3.4

Q5

Vol = 108



$h = \frac{108}{x^2}$

(i) $V = L \times W \times h$

$108 = (x)(x)(h)$

$\frac{108}{x^2} = h$

open at the top

(ii) $SA = \cancel{2}(x)(x) + 2(x)\left(\frac{108}{x^2}\right) + 2(x)\left(\frac{108}{x^2}\right)$

$= \cancel{2}x^2 + \frac{216}{x} + \frac{216}{x}$

$SA = \cancel{2}x^2 + \frac{432}{x}$

(iii) Min $\Rightarrow \frac{dA}{dx} = 0$

$\frac{dA}{dx} = 2x - 432x^{-2} = 2x - \frac{432}{x^2} = 0$

$2x^3 = 432$

$x^3 = 216$

$x = 6.$

Show is a Min.

$\frac{d^2A}{dx^2} = 2 + 864x^{-3}$

at $x = 6$ $\quad 2 + 864(6)^{-3} = 6 > 0$

$\Rightarrow$ Min.

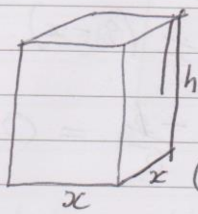
Hence dimensions are Length = 6

Width = 6

Height $= \frac{108}{6^2} = 3$.

Ex 3.4

Q7



$SA = 54cm^2$

(i) $2(x)(x) + 2(x)(h) + 2(x)(h) = 54$

$2x^2 + 4xh = 54.$

$h = \dfrac{54 - 2x^2}{4x}$

$h = \dfrac{27 - x^2}{2x}$

(ii) $V = (x)(x)\left(\dfrac{27 - x^2}{2x}\right)$

$\dfrac{x^{\cancel{2}}(27 - x^2)}{\cancel{2x}} = \dfrac{27x - x^3}{2} = \frac{1}{2}(27x - x^3)$

(iii) $max \Rightarrow dy/dx = 0.$

$V = \frac{1}{2}(27x - x^3)$ (Product rule)

$\dfrac{dv}{dx} = \frac{1}{2}(27 - 3x^2) + (27x - x^3)(0)$

$\frac{1}{2}(27 - 3x^2) = 0$

$27 - 3x^2 = 0$

$3x^2 = 27$ (−3 Not Poss for length)

$x^2 = 9$

$x = 3$

check is a max.

$\dfrac{dv}{dx} = \dfrac{1}{2}(27 - 3x^2)$

$\dfrac{d^2V}{dx^2} = \frac{1}{2}(-6x) + (27 - 3x^2)(0)$

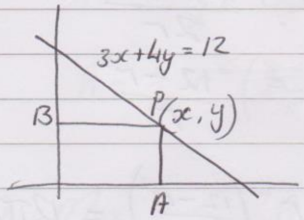
$= \frac{1}{2}(-6x) = -3x$ at $x = 3$

$-3(3) = -9 < 0 \Rightarrow Max.$

$$\therefore h = \frac{27 - x^2}{2x} = \frac{27 - (3)^2}{2(3)} = \frac{18}{6} = 3$$

$$\Rightarrow V = (3)(3)(3) = 27 cm^3$$

Q8



(i)
$$3x + 4y = 12$$
$$4y = 12 - 3x$$
$$y = \frac{12 - 3x}{4}$$

$$P\left(x, \frac{12 - 3x}{4}\right)$$

(ii) $$\text{Area} = (x)\left(\frac{12 - 3x}{4}\right) = \frac{1}{4}(12x - 3x^2)$$

(iii) $$\text{Max} \Rightarrow \frac{dA}{dx} = 0$$

$$A = \left(\frac{1}{4}\right)(12x - 3x^2)$$
$$\frac{dA}{dx} = \frac{1}{4}(12 - 6x) + (12x - 3x^2)(0)$$

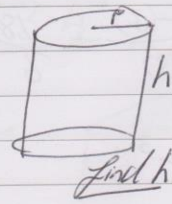
$$\boxed{\frac{dA}{dx} = 0}$$
$$\frac{1}{4}(12 - 6x) = 0$$
$$12 - 6x = 0$$
$$6x = 12$$
$$x = 2$$

Check Max $\quad \frac{dA}{dx} = \frac{1}{4}(12 - 6x)$

$$d^2A/dx^2 = \frac{1}{2}(-6) + (12 - 6x)(0) = -3 < 0 \Rightarrow \text{Max}$$

$$\Rightarrow A = \left(\frac{1}{4}\right)(12x - 3x^2) = \left(\frac{1}{4}\right)(12(2) - 3(2)^2)$$
$$= \frac{1}{4}(24 - 12) = \frac{1}{4}(12)$$
$$= 3 \text{ sq Units.}$$

Q9



Find h

$SA = 24\pi \text{ cm}^2$ $\qquad V = \pi r^2 h.$

$$\cancel{2\pi} r^2 + \cancel{2\pi} rh = 24\cancel{\pi}$$

$$2r^2 + 2rh = 24$$

$$h = \frac{24 - 2r^2}{2r}$$

$$h = \frac{12 - r^2}{r}$$

$$Vol = \pi r^2 h = \pi r^{\cancel{2}}\left(\frac{12 - r^2}{\cancel{r}}\right) = 12\pi r - \pi r^3$$

$\text{Max} \Rightarrow \frac{dV}{dr} = 0$

$$V = 12\pi r - \pi r^3$$

$$\frac{dV}{dr} = 12\pi - 3\pi r^2$$

$$12\pi - 3r^2\pi = 0 \ (\div \pi)$$

$$12 - 3r^2 = 0$$

$$12 = 3r^2$$

$$4 = r^2$$

$$2 = r.$$

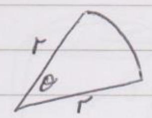
Check Max

$$\frac{dV}{dr} = 12\pi - 3\pi r^2$$

$$\frac{d^2V}{dr^2} = -6\pi r \quad \text{at } r = 2 \Rightarrow -6\pi(2)$$

$$= -12\pi < 0 \Rightarrow \text{Max}$$

Q11



(i) $P = 8$. $\quad P = r + r + r\theta$

$$2r + r\theta = 8$$
$$\theta = \frac{8-2r}{r}$$

(ii) $A = \frac{1}{2}r^2\theta$

$$A = \frac{1}{2}r^2\left(\frac{8-2r}{r}\right) = \frac{8r-2r^2}{2} = 4r - r^2$$

(iii) Max:
$$A = 4r - r^2$$
$$\frac{dA}{dr} = 4 - 2r$$

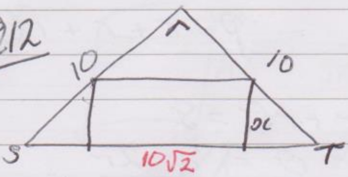
$\frac{dA}{dr} = 0 \quad 4 - 2r = 0$
$$2r = 4$$
$$r = 2.$$

check Max

$$\frac{dA}{dr} = 4 - 2r$$
$$\frac{d^2A}{dr^2} = -2 < 0 \Rightarrow \text{Max}.$$

$r = 2 \Rightarrow A = 4r - r^2$
$$= 4(2) - (2)^2$$
$$= 8 - 4$$
$$= 4\,m^2$$

Q12



(a)

(i) ST $\Rightarrow$ $|ST|^2 = 10^2 + 10^2$

$|ST|^2 = 200$

$ST = \sqrt{200}$

$ST = 10\sqrt{2}$.

(ii) Width of rec $= x$ Length of rec $= 10\sqrt{2} - 2x$

$$\text{Area} = (10\sqrt{2} - 2x)(x) = (10\sqrt{2})x - 2x^2$$

(b) Max $\Rightarrow \frac{dA}{dx} = 0$.

$$A = 10\sqrt{2}x - 2x^2$$

$$\frac{dA}{dx} = 10\sqrt{2} - 4x$$

$\frac{dA}{dx} = 0$ $\quad 10\sqrt{2} - 4x = 0$

$\frac{10\sqrt{2}}{4} = x$ width $= \frac{5\sqrt{2}}{2}$

length $= 10\sqrt{2} - 2x = 10\sqrt{2} - 2\left(\frac{10\sqrt{2}}{\cancel{4}\,2}\right)$

$= 10\sqrt{2} - 5\sqrt{2}$

$= 5\sqrt{2}$ length

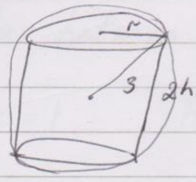
Check Max

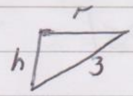
$$\frac{dA}{dx} = 10\sqrt{2} - 4x$$

$$\frac{d^2A}{dx^2} = -4 < 0 \Rightarrow \text{Max}$$

Length $= 5\sqrt{2}$ width $= \frac{5\sqrt{2}}{2}$

Q13





$3^2 = h^2 + r^2$

$9 = h^2 + r^2$

$9 - h^2 = r^2$

$\sqrt{9-h^2} = r$

$V = \pi r^2 h$

$V = \pi\left(\sqrt{9-h^2}\right)^2 (2h) = \pi(9-h^2)(2h) = 18h\pi - 2h^3\pi$

$= 2\pi h(9-h^2)$

Max $\Rightarrow \frac{dV}{dh} = 0$ $\quad V = 18h\pi - 2h^3\pi$

$\frac{dV}{dh} = 18\pi - 6\pi h^2$

$\frac{dV}{dh} = 0 \quad 18\pi - 6\pi h^2 = 0 \quad (\div 6\pi)$

$3 - h^2 = 0$

$3 = h^2$

$\sqrt{3} = h$

$r = \sqrt{9-h^2}$

$= \sqrt{9-\sqrt{3}^2}$

$= \sqrt{9-3}$

$r = \sqrt{6}$

check Max

$\frac{dV}{dx} = 18\pi - 6\pi h^2$

$\frac{d^2V}{dx^2} = -12\pi h$

at $h = \sqrt{3}$ $\quad -12\pi(\sqrt{3}) = -12\sqrt{3}\pi < 0 \Rightarrow$ Max.

$\therefore$ Vol $= 2\pi h(9-h^2)$ at $h = \sqrt{3}$

$= 2\pi\sqrt{3}(9-\sqrt{3}^2)$

$= 2\sqrt{3}\pi(9-3)$

$= 2\sqrt{3}\pi(6)$

Vol $= 12\sqrt{3}\pi$ cm^3

Q14 (a)(i) $|PS| = 6 - x$

$|RS| = 12 - \frac{8}{x}$

(ii) $A = (6-x)\left(12 - \frac{8}{x}\right)$

$= 72 - \frac{48}{x} - 12x + \cancel{\frac{8x}{x}}$

$A = 80 - \frac{48}{x} - 12x$.

$= 80 - 12x - \frac{48}{x}$

(b) Max or Min $\Rightarrow \frac{dA}{dx} = 0$.

$A = 80 - 12x - 48x^{-1}$

$\frac{dA}{dx} = -12 + 48x^{-2}$

$\frac{dA}{dx} = 0 \qquad -12 + \frac{48}{x^2} = 0 \quad (\times x^2)$

$-12x^2 + 48 = 0$

$12x^2 = 48$

$x^2 = 4$

$x = 2$ (from Diag $x \neq -2$)

$\frac{dA}{dx} = -12 + 48x^{-2}$

$\frac{d^2A}{dx^2} = \frac{-96}{x^3}$ at $x = 2$ $\frac{-96}{8} = -12 < 0 \Rightarrow$ Max.

$\therefore$ Max Area $\Rightarrow A = 80 - \frac{48}{x} - 12x$

at $x = 2$ $A = 80 - \frac{48}{2} - 12(2)$

$= 80 - 24 - 24$

$= 32$ sq units

Least Value

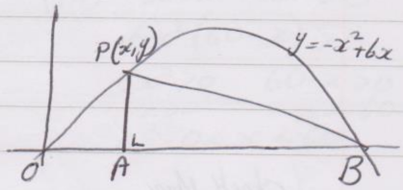
P lies between $x=1$ and $x=4$.

at $x=1$ $\quad A = 80 - 12(1) - \frac{48}{1} = 20$ Min

at $x=4$ $\quad A = 80 - 12(4) - \frac{48}{4} = 20$ Min.

$\Rightarrow$ Max Area $= 32$ sq units and Min Area $= 20$ sq units.

Q15 $\quad y = -x^2 + 6x$



(i) $P\left(x, -x^2+6x\right)$

(ii) $A = \frac{1}{2}$ base $\times$ height. $\quad$ height $= y = \left(-x^2+6x\right)$

find base

$|OA| = x$ $\quad$ at B $y=0 \Rightarrow 0 = -x^2+6x \ (\div x)$

$0 = -x+6$

$\Rightarrow |OB| = 6.$ $\quad x = 6.$

$\therefore |AB| = 6 - x$ base.

$$\text{Area} = \frac{1}{2}(6-x)\left(-x^2+6x\right)$$
$$= \frac{1}{2}\left(-6x^2 + 36x + x^3 - 6x^2\right)$$
$$= \frac{1}{2}\left(x^3 - 12x^2 + 36x\right)$$

$$\text{Area} = \frac{x^3}{2} - 6x^2 + 18x$$

(iii) Max $\Rightarrow \frac{dA}{dx} = 0$

$$A = \frac{x^3}{2} - 6x^2 + 18x$$

$$\frac{dA}{dx} = \frac{3}{2}x^2 - 12x + 18.$$

$\frac{dA}{dx} = 0$

$$\tfrac{3}{2}x^2 - 12x + 18 = 0$$
$$3x^2 - 24x + 36 = 0$$
$$x^2 - 8x + 12 = 0$$
$$(x-2)(x-6) = 0$$
$$\boxed{x = 2} \qquad x = 6.$$ Not Valid as $B = (6,0)$

Check Max

$$\frac{dA}{dx} = \tfrac{3}{2}x^2 - 12x + 18$$
$$\frac{d^2A}{dx^2} = \frac{6}{2}x - 12 \qquad = 3x - 12$$

at $x = 2$ $\quad 3(2) - 12 = -6 < 0$

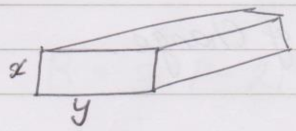
$\Rightarrow$ Max.

$$\text{Area} = \frac{x^3}{2} - 6x^2 + 18x$$

at $x = 2$ $\quad \frac{(2)^3}{2} - 6(2)^2 + 18(2)$

$$4 - 24 + 36 \quad = 16 \text{ sq units}$$

Q16



$S = 5x^2y$

$P = 120.$

(i) $\Rightarrow 2x + 2y = 120$

$y = \frac{120 - 2x}{2}$

$y = 60 - x$

(ii) $S = 5x^2y$

$S = 5x^2(60 - x)$

$= 300x^2 - 5x^3$

(iii) $300x^2 - 5x^3 > 0$

$5x^2(60 - x) > 0$

$5x^2 > 0 \quad 60 - x > 0$

$x > 0 \quad x < 60$

$0 < x < 60$

(iv) $S = 300x^2 - 5x^3$

$\frac{dS}{dx} = 600x - 15x^2$

$\frac{dS}{dx} = 0 \qquad 600x - 15x^2 = 0$

$15x(40 - x) = 0$

$15x = 0 \qquad 40 - x = 0$

$x = 0 \qquad \underline{x = 40}$

Not Valid

$\frac{dS}{dx} = 600x - 15x^2$

$\frac{d^2S}{dx^2} = 600 - 30x$

at $x = 40$

$600 - 30(40) = 600 - 1200$

$= -600 < 0 \Rightarrow$ Max.

$\Rightarrow$ Max Strength at $x = 40$.

$y = 60 - x$

$y = 60 - 40$

$y = 20$

Max S at $x = 40$ and $y = 20$

(v) $x < 19 \Rightarrow$ Max $S = 5(19)^2(60 - 19)$

$= 5(19)^2(41)$

$= 74005$