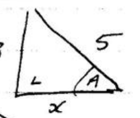
Solutions to ex 5.3 – Text and Tests 4 Trigonometry 2

Ex 5·3

Q1 $\quad \sin A = 3/5$



$\Rightarrow x = 4$

$\Rightarrow \cos A = \frac{4}{5}$

$\tan A = 3/4$

(i) $\sin 2A = 2\sin A\cos A$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A$

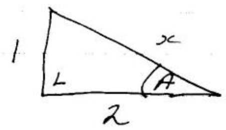
$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = 7/25$

(iii) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$= \dfrac{2(3/4)}{1-(3/4)^2} = \dfrac{6/4}{1-9/16} = \dfrac{6/4}{7/16}$

$= \frac{6}{4} \times \frac{16}{7} = \frac{24}{7}$

Q2 $\quad \tan A = 1/2$



$\Rightarrow x = \sqrt{5}$

$\Rightarrow \sin A = 1/\sqrt{5}$

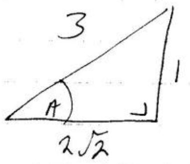
$\cos A = 2/\sqrt{5}$

(i) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A} = \dfrac{2(\frac{1}{2})}{1-(1/2)^2} = \dfrac{1}{1-1/4} = \dfrac{1}{3/4} = 4/3$

(ii) $\sin 2A = 2\sin A\cos A$

$= 2\left(1/\sqrt{5}\right)\left(2/\sqrt{5}\right) = \frac{4}{5}$

Q3



$x^2 = (2\sqrt{2})^2 + (1)^2$

$x^2 = 8 + 1 = 9$

$x = 3$

$\cos A = 2\sqrt{2}/3$

$\sin A = 1/3$

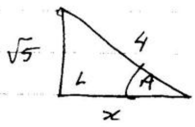
$\tan A = \frac{1}{2\sqrt{2}}$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= \left(2\sqrt{2}/3\right)^2 - \left(\frac{1}{3}\right)^2$$
$$= 8/9 - \frac{1}{9} = \boxed{7/9}$$

Q4.

$\cos 2A = 3/8$. find $\sin A$ + $\cos A$.

$$\cos 2A = 1 - 2\sin^2 A.$$
$$3/8 = 1 - 2\sin^2 A$$
$$2\sin^2 A = 5/8$$
$$\sin^2 A = 5/16$$
$$\sin A = \sqrt{5/16} = \boxed{\frac{\sqrt{5}}{4}}.$$



$4^2 = \sqrt{5}^2 + x^2$

$16 - 5 = x^2$

$\sqrt{11} = x$ $\Rightarrow \cos A = \boxed{\frac{\sqrt{11}}{4}}$

or

$$\cos 2A = 2\cos^2 A - 1$$
$$3/8 = 2\cos^2 A - 1$$
$$\frac{11}{8} = 2\cos^2 A$$
$$\frac{11}{16} = \cos^2 A$$

$$\cos A = \sqrt{\frac{11}{16}} = \frac{\sqrt{11}}{4}$$

Q5 $\quad Sin2A = 2SinACosA$

(i) $\quad 2 Sin15 Cos15 = Sin2A$

$\Rightarrow A = 30°$

$\Rightarrow Sin30 = \frac{1}{2}$

$Cos2A = Cos^2A - Sin^2A$

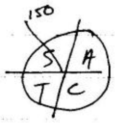
$Cos^2 22\frac{1}{2} - Sin^2 22\frac{1}{2}$

$= Cos 45$

$= \frac{1}{\sqrt{2}}$

(ii) $\quad 2 Sin 75 Cos 75$

$\Rightarrow A = 150°$



ref angle = 30

$Sin 30 = \frac{1}{2}$

Q6 $\quad \frac{2 Tan 22\frac{1}{2}}{1 - Tan^2 22\frac{1}{2}} = 2 Tan A$

$\Rightarrow A = 45.$

$Tan 45 = 1.$

Q7 Prove

$Cos3A = 4Cos^3A - 3CosA.$

$Cos3A = Cos(2A+A)$

$Cos(2A+A) = Cos2ACosA - Sin2ASinA$

$= (2Cos^2A - 1)CosA - (2SinACosA)SinA.$

$= 2Cos^3A - CosA - 2\underline{Sin^2A}CosA$

$= 2Cos^3A - CosA - 2(1 - Cos^2A)CosA$

$= 2Cos^3A - CosA - 2(CosA - Cos^3A)$

$= 2Cos^3A - CosA - 2CosA + Cos^3A$

$= 3Cos^3A - 3CosA$

QED

Q8 Prove

(i) $(\sin A + \cos A)^2 = 1 + \sin 2A$

$\sin^2 A + 2\sin A\cos A + \cos^2 A$

$1 + \sin 2A$ QED.

(ii) $\dfrac{\cos 2A}{\cos A + \sin A} = \cos A - \sin A$

$\dfrac{\cos^2 A - \sin^2 A}{\cos A + \sin A}$

$\dfrac{\cancel{(\cos A + \sin A)}(\cos A - \sin A)}{\cancel{\cos A + \sin A}} = \cos A - \sin A$

QED.

Q9 show $1 - (\cos x - \sin x)^2 = \sin 2x$

$1 - [\cos^2 A - 2\cos x \sin x + \sin^2 x]$

$1 - \cos^2 A + 2\cos x \sin x - \sin^2 x$

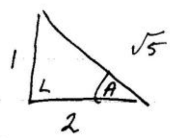
$1 - \cos^2 A - \sin^2 A \quad + 2\cos x \sin x$

$1 - (\cos^2 A + \sin^2 A) \quad + 2\cos x \sin x$

$1 - 1 \quad + 2\cos x \sin x$

$= \sin 2x$ QED.

Q10 $\quad \tan A = \frac{1}{2}$



$x^2 = 1^2 + 2^2$

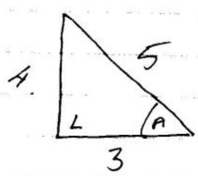
$x = \sqrt{5}$

Find $\tan 2A$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2(\frac{1}{2})}{1-(\frac{1}{2})^2} = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}} = \boxed{\frac{4}{3}}$$

Q11 $\quad \cos A = 3/5$



$\cancel{5}^2 = 3^2 + x^2$

$x = 4.$

$\Rightarrow \sin A = 4/5$

$\tan A = 4/3$

(i) $$\sin 2A = 2\sin A\cos A$$
$$= 2\left(\tfrac{4}{5}\right)\left(\tfrac{3}{5}\right)$$
$$= \frac{24}{25}$$

(ii) $$\cos 2A = \cos^2 A - \sin^2 A$$
$$= \left(\tfrac{3}{5}\right)^2 - \left(\tfrac{4}{5}\right)^2$$
$$= \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}$$

Q12 Prove $\dfrac{1 - \text{Cos}\,2A}{\text{Sin}\,2A} = \text{Tan}\,A$

$$\cancel{\frac{1 - (\text{Cos}^2 A - \text{Sin}^2 A)}{2\,\text{Sin}\,A\,\text{Cos}\,A}}$$

$$\cancel{\frac{1 - (\text{Cos}^2 A + \text{Sin}^2 A)}{2\,\text{Sin}\,A\,\text{Cos}\,A}}$$

$$\cancel{1 - 1}$$

$$\frac{1 - (1 - 2\,\text{Sin}^2 A)}{2\,\text{Sin}\,A\,\text{Cos}\,A}$$

$$\frac{\cancel{1-1} + 2\,\overset{\text{Sin}\,A}{\cancel{\text{Sin}^2 A}}}{\cancel{2}\,\cancel{\text{Sin}\,A}\,\text{Cos}\,A}$$

$$= \frac{\text{Sin}\,A}{\text{Cos}\,A} = \text{Tan}\,A.$$

Q13 Show $\dfrac{2\,\text{Tan}\,A}{1 + \text{Tan}^2 A} = \text{Sin}\,2A$

$$\frac{2\left(\frac{\text{Sin}\,A}{\text{Cos}\,A}\right)}{1 + \frac{\text{Sin}^2 A}{\text{Cos}^2 A}}.$$

$$\frac{2\left(\frac{\text{Sin}\,A}{\text{Cos}\,A}\right)}{\frac{\text{Cos}^2 A + \text{Sin}^2 A}{\text{Cos}^2 A}}$$

$$\frac{2\,\frac{\text{Sin}\,A}{\text{Cos}\,A}}{\frac{1}{\text{Cos}\,A}}$$

$$2\,\frac{\text{Sin}\,A}{\cancel{\text{Cos}\,A}} \times \frac{\cancel{\text{Cos}\,A}}{1}$$

$$= 2\,\text{Sin}\,A$$

Q14 $\quad \tan 2\theta = \frac{4}{3}$

$$\frac{2\tan A}{1-\tan^2 A} = \frac{4}{3}$$

$$6\tan A = 4(1-\tan^2 A)$$
$$6\tan A = 4 - 4\tan^2 A$$
$$4\tan^2 A + 6\tan A - 4 = 0$$
$$(\div 2) \quad 2\tan^2 A + 3\tan A - 2 = 0$$

$$(2\tan A - 1)(\tan A + 2) = 0$$

$$2\tan A - 1 = 0 \qquad \tan A + 2 = 0$$
$$2\tan A = 1$$
$$\tan A = \tfrac{1}{2} \qquad \tan A = -2.$$

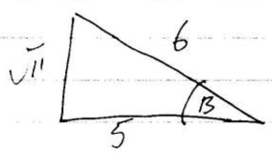
Q15 (i) $$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 2B}{5} = \frac{\sin B}{3}$$
$$\sin 2\beta = \frac{5\sin\beta}{3}$$

(ii) $$\tan\beta = \frac{\sin\beta}{\cos\beta}$$

$$\sin 2\beta = 2\sin\beta\cos\beta$$
$$\frac{5\cancel{\sin\beta}}{3} = 2\cancel{\sin\beta}\cos\beta$$
$$\frac{5}{6} = \cos\beta$$

$$\tan\beta = \frac{\sqrt{11}/6}{5/6}$$
$$\frac{\sqrt{11}}{\cancel{6}} \times \frac{\cancel{6}}{5}$$
$$\tan\beta = \frac{\sqrt{11}}{5}$$



$$6^2 = 5^2 + x^2$$
$$\sqrt{11} = x$$
$$\Rightarrow \sin\beta = \frac{\sqrt{11}}{6}$$

Q16 (i) $\quad \operatorname{Tan} A = \frac{4}{3} \qquad \operatorname{Tan}(A+B) = -1$

$$\operatorname{Tan}(A+B) = \frac{\operatorname{Tan} A - \operatorname{Tan} B}{1 + \operatorname{Tan} A \operatorname{Tan} B}$$

$$-1 = \frac{4/3 - \operatorname{Tan} B}{1 + \frac{4}{3}\operatorname{Tan} B}$$

$$-1 - \frac{4}{3}\operatorname{Tan} B = \frac{4}{3} - \operatorname{Tan} B$$

$$+\operatorname{Tan} \beta - \frac{4}{3}\operatorname{Tan} \beta = +\frac{4}{3} + 1$$

$$-\frac{1}{3}\operatorname{Tan} B = +\frac{7}{3}$$

$$\operatorname{Tan} \beta = +\frac{7}{3} \times \frac{-3}{1}$$

$$\operatorname{Tan} \beta = -7$$

(ii) $\quad \operatorname{Sin} 2\beta = \frac{2\operatorname{Tan} \beta}{1 + \operatorname{Tan}^2 \beta}$

$$= \frac{2(7)}{1 + (7)^2} = \frac{14}{50} = \frac{7}{25}$$

Q17 (i) $\dfrac{\sin 2A}{1+\cos 2A} = \tan A$

$$\frac{2\sin A\cos A}{\cancel{\sin^2 A}+\cos^2 A + \cos^2 - \cancel{\sin^2 A}}$$

$$\frac{\cancel{2}\sin A\cos A}{\cancel{2}\cos^{\cancel{2}} A}$$

$\cos A$

$$\frac{\sin A}{\cos A} = \tan A \qquad [RHS]$$

(ii) $\tan 22\tfrac{1}{2} = \sqrt{2}-1$

$$\tan 22\tfrac{1}{2} = \frac{\sin 2(22\tfrac{1}{2})}{1+\cos 2(22\tfrac{1}{2})} = \frac{\sin 45}{1+\cos 45}$$

$$= \frac{1/\sqrt{2}}{1+1/\sqrt{2}} = (\cancel{x}\ \sqrt{2})$$

$$\frac{1}{\sqrt{2}+1} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{\cancel{\sqrt{2}} - 2 + 1 - \cancel{\sqrt{2}}}$$

$$= \frac{1-\sqrt{2}}{-1} = \sqrt{2}-1$$

Q18 $\cos 2A = 1 - 2\sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

(i) $\cos 4A = \cos(2A + 2A)$

$= 1 - 2\sin^2 2A$

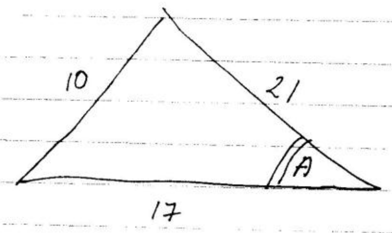
(ii) $\cos 4A = 2\cos^2 2A - 1$

Show $\dfrac{1 - \cos 4A}{1 + \cos 4A} = \tan^2 2A$

$\dfrac{1 - (1 - 2\sin^2 2A)}{\cancel{1} + 2\cos^2 2A \cancel{-1}} = \dfrac{2\sin^2 2A}{\cancel{2}\cos^2 2A}$

$= \dfrac{\sin^2 2A}{\cos^2 2A} = \tan^2 2A.$

Q19



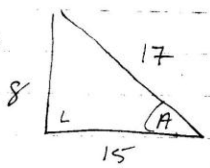
(i)

Cosine Rule:

$$(10)^2 = (21)^2 + (17)^2 - 2(21)(17)\cos A$$
$$100 = 441 + 289 - 714\cos A$$
$$100 = 730 - 714\cos A$$
$$100 - 730 = -714\cos A$$
$$(+)630 = (+)714\cos A$$
$$\frac{630}{714} = \cos A$$

$$\frac{15}{17} = \cos A$$

(ii) $\tan \frac{A}{2}$.



$17^2 = 15^2 + x^2$

$x = 8$

$\Rightarrow \tan A = \frac{8}{15}$.

$$\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$$

$$\frac{8}{15} = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$$

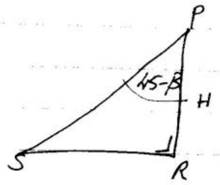
$$8 - 8\tan^2\frac{A}{2} = 30\tan A/2$$

$$8\tan^2 A/2 + 30\tan A/2 - 8 = 0 \qquad (\div 2)$$
$$4\tan^2 A/2 + 15\tan A/2 - 4 = 0$$
$$(4\tan A/2 - 1)(\tan A/2 + 4) = 0$$

Ans: $\tan\frac{A}{2} = +\frac{1}{4}$ or $\tan\frac{A}{2} = -4$ ✗

Q20

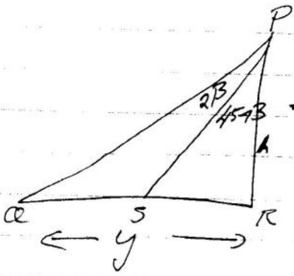
(i) $|SR| = h \tan(45-\beta)$



$\tan(45-\beta) = \frac{|SR|}{h}$

$h \tan(45-\beta) = |SR|$ QED.

(ii) $|QS| = 2h \tan 2\beta.$



$\Rightarrow 45+\beta.$

$\tan(45+\beta) = \frac{y}{h}$

$h \tan(45+\beta) = y$

$\boxed{\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta}}$

$|QS| = h \tan(45+\beta) - h \tan(45-\beta)$

$= h[\tan 45+\beta - \tan 45-\beta]$

$= h\left[\frac{\tan 45 + \tan\beta}{1-\tan 45 \tan\beta} - \frac{\tan 45 - \tan\beta}{1+\tan 45 \tan\beta}\right]$

$= h\left[\frac{1+\tan\beta}{1-\tan\beta} - \frac{1-\tan\beta}{1+\tan\beta}\right]$

$= h\left[\frac{(1+\tan\beta)^2 - (1-\tan\beta)^2}{(1-\tan\beta)(1+\tan\beta)}\right]$

$= h\left[\frac{1+2\tan\beta+\tan^2\beta - 1 + 2\tan\beta - \tan^2\beta}{1+\tan^2\beta}\right]$

$= h\left[\frac{4\tan\beta}{1-\tan^2\beta}\right] = 2h\left(\frac{\tan\beta}{1-\tan^2\beta}\right)$

$= 2h\tan 2\beta$

QED