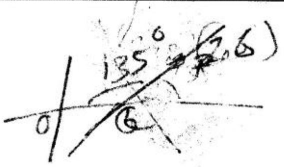
## Solutions to Paper 2 - Sample Paper A

| Section A | Concepts and Skills | 150 marks |
|---|---|---|

Answer **all six questions** from this section



### Question 1 **(25 marks)**

The line $l$ passes through P(2,6) and cuts the y-axis at Q, where $|\angle PQO| = 135°$ and O is the origin.

(i) Find the slope of $l$ and plot $l$ on the coordinate plane.

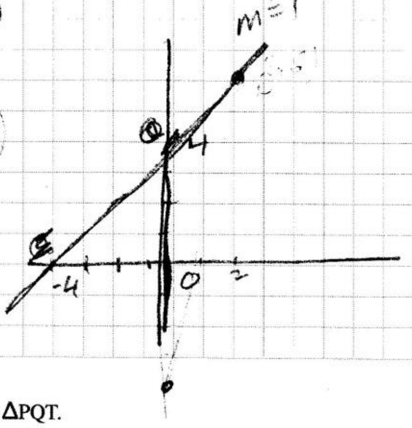
$180 - 135 = 45°$ $\qquad m = \tan\theta = \tan 45 = 1$

eqn: $y - y_1 = m(x - x_1)$

$y - 6 = 1(x-2)$

$y = x + 4$

$(0,4)$ or $(-4,0)$



(ii) If T is the point (0,–4), show that 8 is the area of the $\Delta$PQT.

P Q T

$(2,6)$ $(0,4)$ $(0,-4)$

$(0,0)$ $(-2,-2)$ $(-2,-10)$

$$\text{Area} = \frac{1}{2}\left|(-2)(-10) - (-2)(-2)\right|$$
$$= \frac{1}{2}\left|20 - 4\right|$$
$$= \frac{1}{2}\left|16\right|$$
$$= 8 \text{ u}^2$$

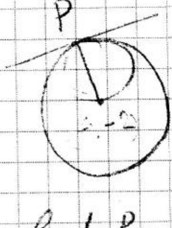
**Question 2** $(4, -2)$ $r = \sqrt{4^2+2^2+5}$ $= 5$ **(25 marks)**

The equation of circle c is $x^2 + y^2 - 8x + 4y - 5 = 0$.

A circle k touches circle c internally and passes through the centre of circle c. ⇒ radius k = diam c

If $3x - 4y + 5 = 0$ is the tangent common to both circles, find the equation of circle k.

require centre + radius $r = 2 \cdot 5$



$\perp$ dis from $(4, -2)$ to $3x - 4y + 5 = 0$ is 5

$3(4) - 4(2)$

find P, pt n of line + circle

$(3x + 5) = 4y \qquad y = \frac{3x+5}{4}$

$x^2 + y^2 - 8x + 4y - 5 = 0$

$x^2 + \left(\frac{3x+5}{4}\right)^2 - 8x + 3x + 5 - 5 = 0$

$x^2 + \frac{9x^2 + 30x + 25}{16} - 5x = 0$

$16x^2 + 9x^2 + 30x + 25 - 80x = 0$

$25x^2 - 50x + 25 = 0$

$x^2 - 2x + 1 = 0$

$(x - 1)(x - 1) = 0$

$x = 1$

$y = \frac{3(1) + 5}{4} = 2$

P $(1, 2)$

Mid pt $(1, 2)$ + $(4, -2)$

$\left(\frac{5}{2}, 0\right)$

centre $\left(\frac{5}{2}, 0\right)$ $r = \frac{5}{2}$

eqn $\left(x - \frac{5}{2}\right)^2 + y^2 = \frac{25}{4}$

## Question 3 **(25 marks)**

(a) State whether the following statements are true or false and explain your answer in each case.
If you do not agree with the answer work out the correct probability.

(i) The probability of getting exactly 1 head in two tosses of a coin is ½. Hence the probability of getting **exactly** 2 tails in 4 tosses of a coin is ½.

T  $P(h)$ : $H\ T$ or $T\ H$

$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

F  $P(2T) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = 0{\cdot}375$

(ii) In a true/false quiz with 8 questions you are certain to get 4 correct if you just guess.

F. → all could be wrong.

$\binom{8}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^4 = 0{\cdot}273\ldots$ = Prob of 4 correct

(b) To win a lottery game, one must choose the winning set of six numbers in any order from the set of all the whole numbers from 1 to 45 inclusive.

Calculate the probability of

(i) Matching the six winning numbers

$\binom{45}{6} = 8145060$

$P(\text{of the } 6) = \frac{1}{8145060}$

(ii) Matching 5 of the winning numbers

$\binom{6}{5} = 6$ and $\binom{39}{1} = 39$

$P = \frac{6 \times 39}{8145060} = 0{\cdot}0000287$

not bonus No

$\binom{6}{5} = 6$ & $\binom{38}{1} = 38$

$P = \frac{6 \times 38}{8145060} = 0{\cdot}000\ldots$

$0{\cdot}00002\ldots$

(iii) Matching 3 of the winning numbers and having the bonus number

$\binom{6}{3}$ and $\binom{1}{1}$ and $\binom{38}{2}$

$20 \times 1 \times 703 = 14060$

$P = \frac{14060}{8145060} = 0{\cdot}001726$

## Question 4 (25 marks)

The school has a very good basketball team. Liam is the best player. He is 208 cm tall. He takes most of his shots close to the basket and because he is so tall he scores 58% of his shots, each of which is worth 2 points. When Liam is fouled he gets two free shots, approximately 3 metres away from the basket, called foul shots. He only scores 51% of these shots which are worth 1 point each. Coach is concerned that opposing teams seem to be fouling Liam whenever he touches the ball, and he asks you to work out if this could pay off for them in the long run in terms of reducing Liam's scores.

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(i) Calculate the expected value of the number of points Liam scores on one regular shot at the basket.

$E = \sum P(x) . x$

| Pts | $P(x)$ | $P(x) . x$ |
|---|---|---|
| 0 | 0.42 | 0 |
| 1 | 0.58 | 1.16 |

1.16

(ii) Calculate the expected value of the number of points Liam scores when he shoots two foul shots. Assume that all foul shots are independent events.

| $x$ | | $P(x)$ | $P(x) . x$ |
|---|---|---|---|
| 0 | Miss + Miss | $0.49 \times 0.49$ | 0 |
| 1 | Miss + Score | $0.49 \times 0.51$ | 0.2499 |
| 1 | Score + Miss | $0.51 \times 0.49$ | 0.2499 |
| 2 | Score + Score | $0.51 \times 0.51$ | 0.5202 |
| | | | 1.02 |

(iii) Which is better for Liam's team – that he is or is not fouled repeatedly? Give one reason why opposing teams might adopt a policy of fouling Liam and one reason why they should not do so.

- $1.16 > 1.02 \Rightarrow$ better if not fouled
- Should foul as less likely to score
- If keep fouling him gets more practice + improves
also more shots $\Rightarrow$ closer to 0.51 $\Rightarrow$ still scoring more than missing

## Question 5

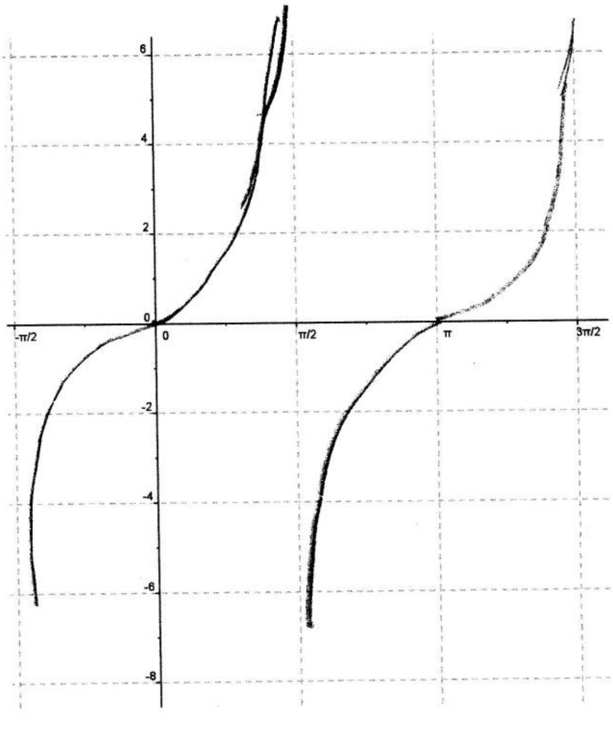
**(25 marks)**

(a) Explain why TanA is not defined for certain values of A and give 3 examples of such values.

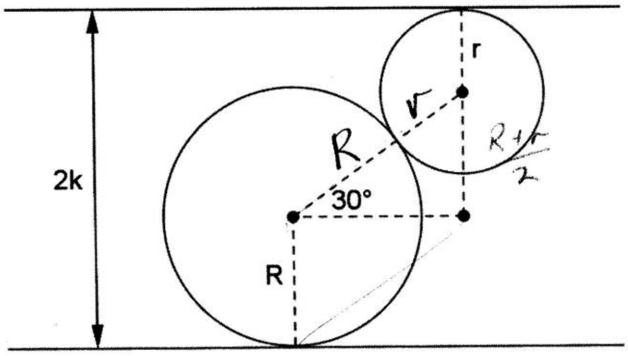
$$\text{Tan } A = \frac{\text{Sin } A}{\text{Cos } A}$$

Tan A not defined when Cos A = 0

Sketch the graph of TanA $\frac{-\pi}{2} \leq A \leq \frac{3\pi}{2}$ in the diagram below.

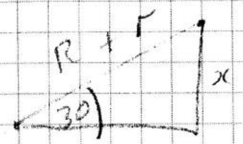


(b) From a strip of tin of width 2k two discs are cut out. One disc touches the top edge of the strip, the second disc touches the bottom edge and both discs touch each other, as in the diagram.




If the line joining the centres of the discs remains at the angle of 30° to the edge of the strip, show that $3(R + r) = 4k$.

$\sin 30 = \frac{1}{2}$




$\sin 30 = \frac{x}{R+r}$

$\frac{1}{2} = \frac{x}{R+r}$

$\frac{R+r}{2} = x$

$r + \frac{R+r}{2} + R = 2k$

$2r + R+r + 2R = 4k$

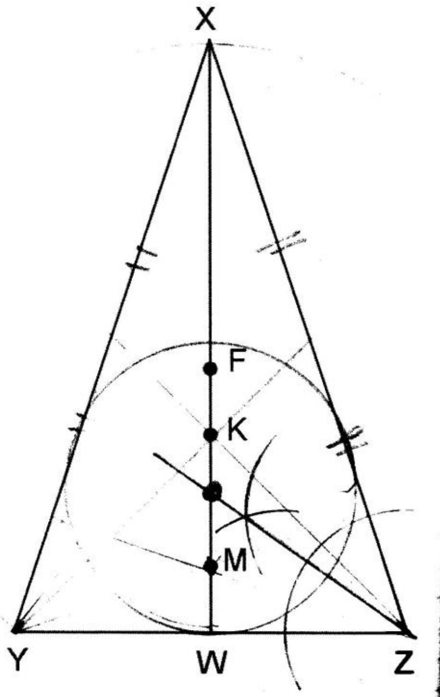
$3r + 3R = 4k$

$3(r+R) = 4k$

## Question 6 (25 marks)

The diagram on the right shows an isosceles triangle XYZ with XW its axis of symmetry.

The centroid, circumcentre and orthocentre of the triangle are all on the line XW and are marked F, K and M, though not in that order.




(i) Using ruler and/or compass, identify clearly which point is which centre.

F is circumcentre (bisect sides)

M is orthocentre vertex ⊥ to opp si

⇒ K is centroid vertex to mid pt opposide

ii) Measure the distances |MK| and |KF| and write down the ratio |MK| : |KF|.

|mK| = 2.4 |KF| = 1.2

2.4 : 1.2

2 : 1

iii) The incentre of the triangle is also on the line XW. Using ruler and compass, construct the incentre and the incircle on the above diagram. Show all construction lines clearly.

Bisect angles

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| Section B | Contexts and Applications | 150 marks |
|---|---|---|

Answer **Question 7, Question 8 and Question 9**

## Question 7

**(75 marks)**

(a) An educational researcher claims that scores achieved using a particular standardised, objective reading test for nine-year-olds in school in Ireland, are approximately normally distributed with a mean score of 100 and a standard deviation of 15. $\sigma = 15$ $\mu = 100$

(i) What is the probability that an individual nine-year-old student taking this test will achieve a score of 108 or less based on the researcher's claim?

$P(x < 108)$

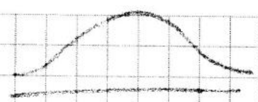
$z = \frac{108 - 100}{15} = 0.5333$

$P(z < 0.5333) = 0.7019$

(ii) What model can be used to describe the distribution of all possible sample means from this population?

Normal distribution

scores are Normal ⇒ means are normal.



(iii) In order to test the researcher's claim, a teacher gives the test to a random sample of 25 nine-year-olds in Ireland. The mean score $\bar{x}$ for the sample is 108. $\bar{x} = 108$ $n = 25$

What is the probability that the mean score for such a random sample is 108 or less?

$P(\bar{x} < 108)$

$z = \frac{108 - 100}{\frac{15}{\sqrt{25}}} = 2.67$

$P(x < 2.67) = 0.99621$

(iv) The teacher decides to use the sample mean to estimate the population mean. Using the sample mean, form a confidence interval for the population mean at the 95% level of confidence.

$\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$

$108 - 1.96\left(\frac{15}{\sqrt{25}}\right) < \mu < 108 + 1.96\left(\frac{15}{\sqrt{25}}\right)$

$108 - 5.88 < \mu < 108 + 5.88$

$102.12 < \mu < 113.88$

(v) The teacher shows the confidence interval to a colleague who asks the following question: "Does this mean that there is a 95% probability that the population mean reading score for nine-year-olds in Irish schools on this standardised test is between the limits of this confidence interval?" Answer this question in a way which will ensure that the colleague understands what a confidence interval means.

for all samples of 25, 95% of them will have a mean between 102.12 & 113.88



(vi) Does the confidence interval lead you to doubt the researcher's claim? Explain.

NO mean score is not within the interval.

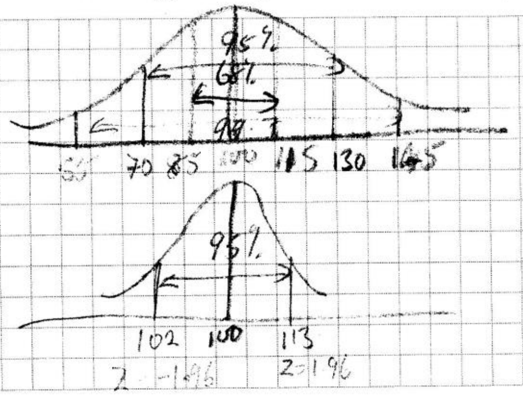
(vii) Conduct a hypothesis test at the 5% level of significance to decide whether or not to reject the researcher's claim. You should start by clearly stating the null hypothesis and the alternative hypothesis and finish by clearly stating your conclusion in context.

$H_0$ $\mu = 100$
$H_1$ $\mu \neq 100$

$$z = \frac{108-100}{15/\sqrt{25}} = 2.67$$

which is greater than 1.96 ⇒ reject $H_0$ & conclude mean ≠ 100

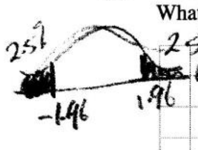
(viii) Draw a sketch of the distribution of test scores for the population. Underneath this draw a sketch of the sampling distribution of means of size $n = 25$ showing the relative positions of the means of both distributions and the approximate position of a score of 108 on both distributions.




$\mu = 100$ $\sigma = 15$.

$\mu = 100$

(ix) What is meant by the p-value for a hypothesis test? Calculate the p-value for the above hypothesis test. What conclusion do you reach based on the p-value?




P value is the Prob that a value is within or outside the 95% interval

$P(z > 2.67) = 2(1-0.99621) = 0.00758$

$\Rightarrow$ reject Ho since P value is less than 5%

(x) What do you mean by a 5% level of significance?

There is a prob of rejecting the Ho even if it is true

(xi) Another teacher carries out the same reading test using a simple random sample but with smaller sample size. Is she justified in applying the normal distribution model? Explain.

Yes since population distribution is Normally distributed.

(b) (i) P(D|P) denotes the conditional probability of "D given P".
Write down an equation to express the relationship between P(P), P(D|P) and P(D∩P).

$$P(D/P) = \frac{P(D \cap P)}{P(P)}$$

(ii) In a certain population, one person in every 200 is known to have a certain disease (D). That is, the probability that a person, chosen at random from the population, has the disease is

$$P(D) = \frac{1}{200} \text{ or } 0.005.$$

A test for the disease is known to be 99% accurate. That is, the probability of testing positive (P) in a person with the disease is 0.99.

1. Find the probability that a person has the disease and tests positive P(D AND P)

P(D) = 0·005 P(P) = 0·99

P(D and P) = 0·005 X 0·99 = 0·00495



2. Find the probability that a person who does not have the disease tests positive P(D'AND P)

P(D') = 0·995 ~~P(P) = 0·99~~ P(P) = 0·01

P(D' and P) = 0·995 x 0·01 = 0·00995

3. Find the probability that a person tests positive P(P)

P(P) = has + Pos or hasnt + Pos

0·00495 + 0·00995 = 0·0149

4. Find the probability that a person from the population who tests positive for this disease actually does have the disease (i.e. P(D|P).

P(D|P) = P(D∩P) / P(P) = 0·00495 / 0·99

= 0·3322

**Question 8** **(50 marks)**

(a) Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

Diagram:

Given:

To Prove:

Construction:

Proof:

(b) An engineer is working in mountainous terrain. She needs to know the distance between two locations E and F which are inaccessible across a deep gorge (see diagram below). She finds two locations C and D on her side of the gorge which are on the same level such that

|CD| = 20 metres.

Using a theodolite, she finds the measure of four angles as follows:

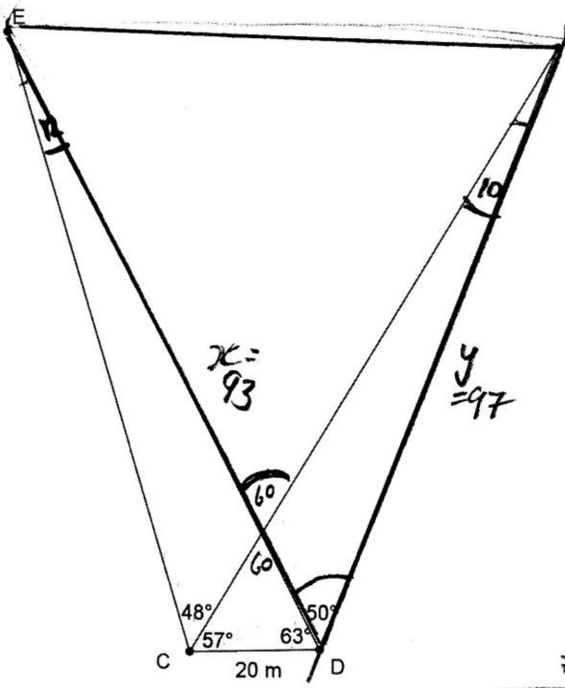
|∠ECF| = 48°

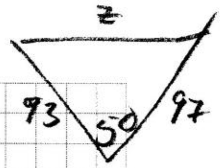
|∠FCD| = 57°

|∠CDE| = 63°

|∠EDF| = 50°

Use this information to find |EF|, showing all your work clearly.




|∠DEC| = 12°

$$\frac{x}{\sin 105} = \frac{20}{\sin 12}$$

$$x = \frac{20}{\sin 12} \times \sin 105$$

$$x = 93\text{m}$$

$$\frac{y}{\sin 57} = \frac{20}{\sin 10}$$

$$y = \frac{20}{\sin 10} \times \sin 57$$

$$y = 97\text{m}$$

$$z^2 = 93^2 + 97^2 - 2(93)(97)\cos 50$$

$$z^2 = 18058 - 11597$$

$$z^2 = 6460.825946$$

$$z = 80.374$$

$$z = 80\text{m}$$

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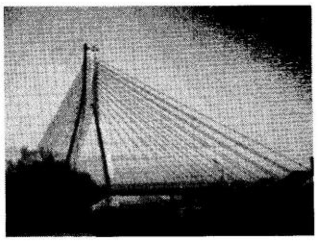
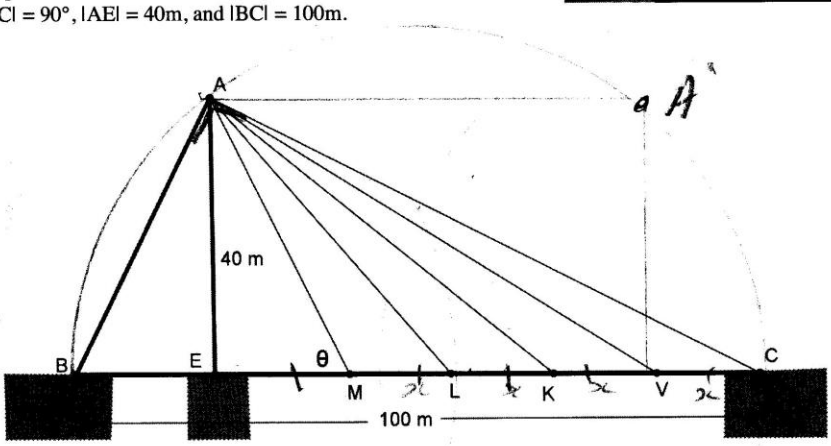
## Question 9 (25 marks)

The diagram below represents a simplified model of a bridge known as a cable-stayed bridge where AE is the supporting tower and BC is the carriageway for traffic. Cable- stayed bridges are very convenient in conditions where there is no good access to the far bank, as they can be built from one side only.



In this particular model,
$|\angle BAC| = 90°$, $|AE| = 40$m, and $|BC| = 100$m.

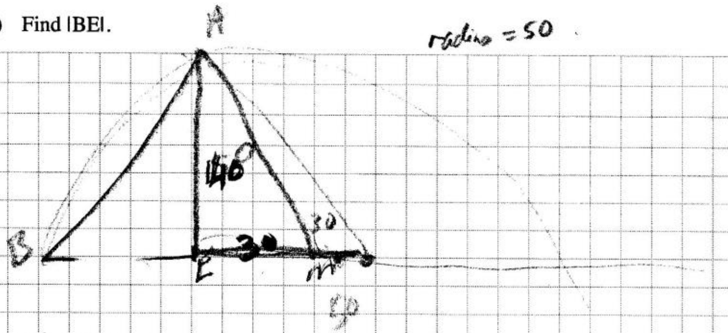


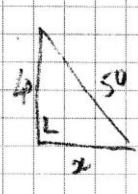

(a) if E is to be nearer to B than to C, use your knowledge of geometry to explain why there is only one location along BC for the tower AE in this model.

If $|\angle BAC| = 90°$ ⇒ BC must be diameter
A & A' only 2 pts ~~atte~~ 40m
Since E must be nearer to B ⇒ A is only pt

(b) Find |BE|.

radius = 50




$50^2 = 40^2 + x^2$

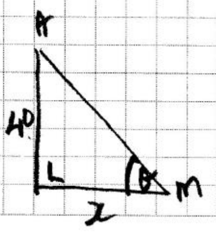
$\sqrt{50^2 - 40^2} = x$

$30 = x$

$\Rightarrow |BE| = 20$

(c) The 5 cables emanating from A meet BC at equal intervals of x metres at the points M, L, K, V and W. **If** tan = 2, where θ is the angle which the shortest cable [AM] makes with BC, find the value of *x*.


$80 \div 5 = 16m = x$




$\tan\theta = 2$

$\theta = \tan^{-1} 2$

$\theta = 63.43$

$\tan\theta = \frac{O}{A}$

$2 = \frac{40}{x}$

$x = \frac{40}{2}$

$x = 20$

$$\begin{array}{r} 80 \\ -20 \\ \hline 60 \end{array} \div 4 = 15m.$$