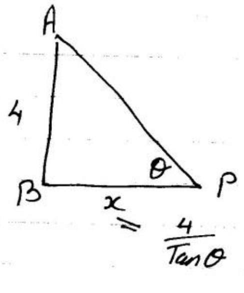
Solutions to homework 28th jan 2013

Test yourself, C questions.

Q2



$$\tan\theta = \frac{4}{x}$$
$$x = \frac{4}{\tan\theta}$$

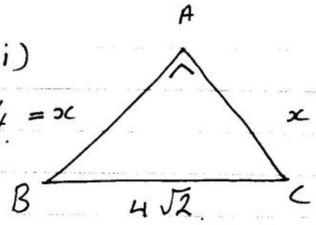
(i)

$$\Rightarrow |PQ| = k - 2\left(\frac{4}{\tan\theta}\right)$$
$$= k - \frac{8}{\tan\theta}$$

(ii)

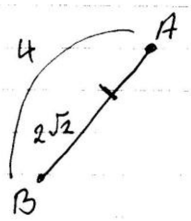
$$12 - 4\sqrt{3} = 12 - \frac{8}{\tan\theta}$$
$$\cancel{12} - 4\sqrt{3} - \cancel{12} = -\frac{8}{\tan\theta}$$
$$+4\sqrt{3} = +\frac{8}{\tan\theta}$$
$$\tan\theta = \frac{8}{4\sqrt{3}}$$
$$\tan\theta = \frac{2}{\sqrt{3}}$$
$$\theta = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$$
$$\theta = 49^\circ$$

Q3 (i)



isos Δ with base angles $45°$ ea

$$(4\sqrt{2})^2 = x^2 + x^2$$
$$32 = 2x^2$$
$$16 = x^2$$
$$4 = x$$



radius $r_1 = 4 - 2\sqrt{2}$

(ii) Shaded Area = Area Δ − area of 3 sectors.

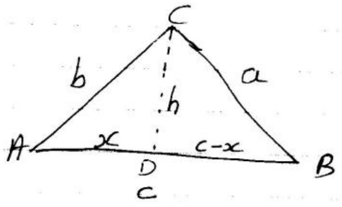
Area ΔABC $= \frac{1}{2}$ base $\times h$
$= \frac{1}{2}(4)(4)$
$= 8$

Area Sector in r_2 $= \frac{45}{360}(\pi)(2\sqrt{2})^2 = \pi$
area Sector in r_3 $= \pi$
area Sector " r_1 $= \frac{90}{360}(\pi)(4 - 2\sqrt{2})^2 =$
$\frac{1}{4}(\pi)(24 - 16\sqrt{2})$
$= \pi(6 - 4\sqrt{2})$

Total $= 2\pi + \pi(6 - 4\sqrt{2})$
$= \pi(2 + 6 - 4\sqrt{2})$
$= (8 - 4\sqrt{2})\pi$

Shaded Area $= 8 - (8 - 4\sqrt{2})\pi$

Q6 (i) Prove $a^2 = b^2 + c^2 - 2bc\cos A$.



In ΔADC : $b^2 = h^2 + x^2 \Rightarrow h^2 = b^2 - x^2$

In ΔCDB : $a^2 = h^2 + (c-x)^2 \Rightarrow h^2 = a^2 - (c-x)^2$

$$b^2 - x^2 = a^2 - (c-x)^2$$

$$b^2 - \cancel{x^2} = a^2 - c^2 + 2cx \cancel{- x^2}$$

But $\cos A = \frac{x}{b} \Rightarrow x = b\cos A$.

$$b^2 + c^2 - 2c\underbrace{b\cos A}_{x} = a^2$$

Q.E.D.

(ii) a b c are consecutive

$\Rightarrow a = a$, $b = (a+1)$ $c = (a+2)$

Sub into Proof above.

$$a^2 = (a+1)^2 + (a+2)^2 - 2(a+2)(a+1)\cos A$$

$$a^2 = a^2 + 2a + 1 + a^2 + 4a + 4 - 2(a^2 + 3a + 2)\cos A$$

$$a^2 = 2a^2 + 6a + 5 - (2a^2 + 6a + 4)\cos A$$

$$a^2 + 6a + 5 = (2a^2 + 6a + 4)\cos A$$

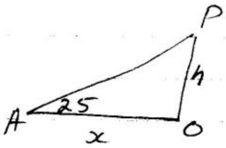
$$\frac{a^2 + 6a + 5}{\underset{2(a^2+3a+2)}{2a^2 + 6a + 4}} = \cos A$$

$$\frac{(a+5)\cancel{(a+1)}}{2(a+2)\cancel{(a+1)}} = \cos A$$

$$\frac{a+5}{2a+4} = \cos A$$

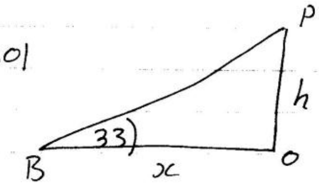
Q.E.D.

Q7 (i) $|AO|$

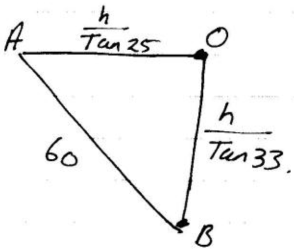


$$\tan 25 = \frac{h}{x}$$
$$x = \frac{h}{\tan 25}$$

(ii) $|BO|$



$$\tan 33 = \frac{h}{x}$$
$$x = \frac{h}{\tan 33}$$



$$60^2 = \left(\frac{h}{\tan 25}\right)^2 + \left(\frac{h}{\tan 33}\right)^2$$

$$3600 = \frac{h^2}{0.217} + \frac{h^2}{0.4217}$$

$$3600(0.217)(0.4217) = h^2(0.4217) + h^2(0.217)$$
$$329.43 = 0.6387\,h^2$$
$$\frac{329.43}{0.6387} = h^2$$
$$515.78 = h^2$$
$$22.7 = h$$