Test Questions B

Q1 (i) Using $\cos 2A = \cos^2 A - \sin^2 A$.
prove $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

$$\cos^2 A = \frac{1}{2}\left[\cos^2 A + \cancel{\sin^2 A} + \cos^2 A - \cancel{\sin^2 A}\right]$$

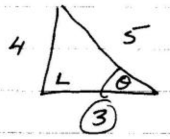
$$= \frac{1}{2}\left(2\cos^2 A\right)$$

$$= \cos^2 A \qquad \text{QED}$$

(ii) $\sin 40 \cos 20 + \cos 40 \sin 20$

$$= \sin(40+20)$$
$$= \sin 60$$
$$= \sqrt{3}/2.$$

Q2 (i) $\sin\theta = 4/5$



$\cos\theta = 3/5$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(3/5\right)^2 - \left(4/5\right)^2 = \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}$$

(ii) $$2\cos^2 A - \cos 2A - 1 = 0$$

$$2\cos^2 A - [\cos^2 A - \sin^2 A] - [\cos^2 A + \sin^2 A]$$

$$2\cos^2 A - \cos^2 A + \sin^2 A - \cos^2 A - \sin^2 A$$

$$= 0$$

QED.

Q3 (i) $2\sin 4\theta \cos 2\theta$
$$= \sin(4\theta + 2\theta) + \sin(4\theta - 2\theta)$$
$$= \sin 6\theta + \sin 2\theta$$

(ii) $$(\cos x + \sin x)^2 + (\cos x - \sin x)^2$$
$$\cos^2 x + \cancel{2\cos x \sin x} + \sin^2 x + \cos^2 x - \cancel{2\cos x \sin x} + \sin^2$$

$$2\cos^2 x + 2\sin^2 x$$

$$2(\cos^2 x + \sin^2 x)$$

$$2(1) \quad = 2.$$

Q4 (i) Prove $\cos(45+\theta) - (\cos 45 - \theta) = -\sqrt{2}\sin\theta$

$$\cos 45\cos\theta - \sin 45\sin\theta - [\cos 45\cos\theta + \sin 45\sin\theta]$$

$$= \tfrac{1}{\sqrt{2}}\cancel{\cos\theta} - \tfrac{1}{\sqrt{2}}\sin\theta - \tfrac{1}{\sqrt{2}}\cancel{\cos\theta} - \tfrac{1}{\sqrt{2}}\sin\theta$$

$$= -2\left(\frac{1}{\sqrt{2}}\sin\theta\right) = \frac{-2}{\sqrt{2}}\sin\theta \qquad \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$$

$$= -\sqrt{2}\sin\theta \qquad \text{RHS}$$

(ii) $\frac{1}{\cos\theta} - \cos\theta = \tan\theta\sin\theta$

$$= \frac{1-\cos^2\theta}{\cos\theta}$$

$$= \frac{1-[1-\sin^2\theta]}{\cos\theta}$$

$$= \frac{\cancel{1}-\cancel{1}+\sin^2\theta}{\cos\theta}$$

$$= \frac{\sin\theta\sin\theta}{\cos\theta} = \tan\theta\sin\theta \qquad \text{RHS}$$

Q5 (i) $\cos^2 15 - \sin^2 15$

$$= \cos 2(15) = \cos 30 = \sqrt{3}/2$$

(ii)

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2.$$

$$\frac{\sin(2\theta+\theta)}{\sin\theta} - \frac{\cos(2\theta+\theta)}{\cos\theta}$$

$$\frac{\sin 2\theta\cos\theta + \cos 2\theta\sin\theta}{\sin\theta} - \frac{\cos 2\theta\cos\theta - \sin 2\theta\sin\theta}{\cos\theta}$$

$$\frac{2\cancel{\sin\theta}\cos\theta\cos\theta + (1-2\sin^2\theta)\cancel{\sin\theta}}{\cancel{\sin\theta}} - \frac{(2\cos^2\theta-1)\cancel{\cos\theta} - 2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}$$

$$2\cos^2\theta + 1 - 2\sin^2\theta - [2\cos^2\theta - 1 - 2\sin^2\theta]$$

$$\cancel{2\cos^2\theta} + 1 - \cancel{2\sin^2\theta} - \cancel{2\cos^2\theta} + 1 + \cancel{2\sin^2\theta}$$

$$= 2 \quad \text{RHS}.$$

Q6 (i) show $\tan 15^\circ = 2 - \sqrt{3}$

$$\tan 15^\circ = \tan(45 - 30)$$

$$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + (1)\frac{1}{\sqrt{3}}}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \quad (\times \sqrt{3}) \quad = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$\frac{(\sqrt{3} - 1)}{(1 + \sqrt{3})} \times \frac{(1 - \sqrt{3})}{(1 - \sqrt{3})} = \frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} - 3} = \frac{-4 + 2\sqrt{3}}{-2}$$

$$= \frac{\cancel{-2}(2 - \sqrt{3})}{\cancel{-2}} = 2 - \sqrt{3} \qquad \text{RHS}$$

(ii) Prove $\dfrac{\cos 5\theta - \cos 3\theta}{\sin 4\theta} = -2 \sin \theta$

$$\frac{-2 \sin \frac{5\theta + 3\theta}{2} \sin \frac{5\theta - 3\theta}{2}}{\sin 4\theta}$$

$$\frac{-2 \cancel{\sin 4\theta} \sin \theta}{\cancel{\sin 4\theta}} = -2 \sin \theta \qquad \text{RHS}$$

Q7 Prove $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ * Formal Proof

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B} \quad \left(\div \text{ Each by } \cos A \cos B\right)$$

$$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\cos B}{\cos B} - \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

QED.

Q8 $\quad A+B = \frac{\pi}{4}$ write Tan A in Terms of Tan B.

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan\left(\frac{\pi}{4}\right) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$1 = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$1 - \tan A \tan B = \tan A + \tan B$$

$$1 - \tan B = \tan A + \tan A \tan B$$

$$1 - \tan B = \tan A (1 + \tan B)$$

$$\frac{1 - \tan B}{1 + \tan B} = \tan A$$

Hence Prove $(1+\tan A)(1+\tan B) = 2$

$$1 + \tan B + \tan A + \tan A \tan B$$ Sub in for Tan A

$$1 + \tan B + \frac{1-\tan B}{1+\tan B} + \frac{1-\tan B}{1+\tan B} \cdot \tan B.$$

$$\frac{1(1+\tan B) + \tan B(1+\tan B) + 1 - \tan B + (1-\tan B)\tan B}{1+\tan B}$$

$$\frac{1 + \cancel{\tan B} + \tan B + \cancel{\tan^2 B} + 1 - \cancel{\tan B} + \tan B - \cancel{\tan^2 B}}{1+\tan B}$$

$$\frac{2 + 2\tan B}{1+\tan B} = \frac{2\cancel{(1+\tan B)}}{\cancel{1+\tan B}} = 2$$

RHS

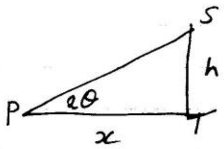
Q9 $\quad \sin 105^\circ - \sin 15^\circ$

$$= 2\cos\frac{105+15}{2}\sin\frac{105-15}{2}$$

$$= 2\cos 60 \sin 45$$

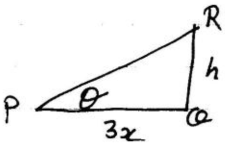
$$= 2\left(\tfrac{1}{2}\right)\left(\tfrac{1}{\sqrt{2}}\right) = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

Q10



$\tan 2\theta = h/x$

$\boxed{x \tan 2\theta = h}$



$\tan\theta = h/3x$

$\boxed{3x \tan\theta = h.}$

equate

$$x\tan 2\theta = 3x\tan\theta \qquad (\div \text{ by } x)$$

$$\tan 2\theta = 3\tan\theta$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = 3\tan\theta$$

$$2\tan\theta = 3\tan\theta\left(1-\tan^2\theta\right)$$

$$2\tan\theta = 3\tan\theta - 3\tan^3\theta \qquad (\div \text{ by } \tan\theta)$$

$$2 = 3 - 3\tan^2\theta$$

$$3\tan^2\theta = 1$$

$$\tan^2\theta = 1/3$$

$$\tan\theta = \sqrt{1/3} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ$$

or $\frac{\pi}{6}$.