Test Questions C

Q1 $\cos(A+B) = \cos A\cos B - \sin A\sin B$

Prove $\cos 2x = 1 - 2\sin^2 x$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - \sin^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \quad \text{RHS}\end{aligned}$$

$$\begin{aligned}\cos 2x = \cos(x+x) &= \cos x\cos x - \sin x\sin x \\ &= \cos^2 x - \sin^2 x \\ &= 1 - \sin^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \quad \text{RHS}\end{aligned}$$

Hence

Prove $\sin 3x = 3\sin x - 4\sin^3 x$

$$\begin{aligned}\sin 3x = \sin(2x+x) &= \sin 2x\cos x + \cos 2x\sin x \\ &(2\sin x\cos x)\cos x + (1-2\sin^2 x)\sin x \\ &2\sin x\cos^2 x + \sin x - 2\sin^3 x \\ &2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x \\ &2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x \\ &3\sin x - 4\sin^3 x \quad \text{RHS.}\end{aligned}$$

Q2 Show $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 2 + 2\cos(A - B)$

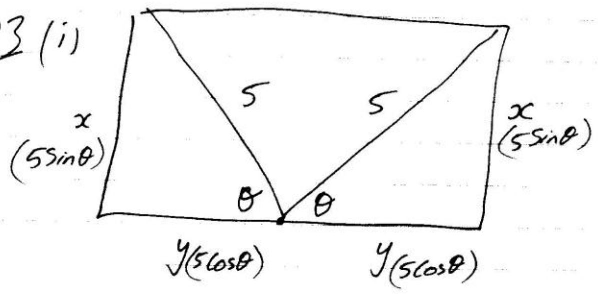
$\cos^2 A + 2\cos A\cos B + \cos^2 B + \sin^2 A + 2\sin A\sin B + \sin^2 B$

$1 + 1 + 2\cos A\cos B + 2\sin A\sin B$

$2 + 2(\cos A\cos B + \sin A\sin B)$

$2 + 2\cos(A-B)$ RHS

Q3 (i)



$\sin\theta = \frac{x}{5}$

$\Rightarrow x = 5\sin\theta$

$\cos\theta = y/5$

$\Rightarrow y = 5\cos\theta$

$P = 2(5\sin\theta) + 2(10\cos\theta)$

$= 10\sin\theta + 20\cos\theta$ QED

(ii) Area $= L \times W = K\sin 2\theta$.

$= (10\cos\theta)(5\sin\theta)$

$= 5[2\cos\theta\sin\theta]$

$= 5[\sin(\theta+\theta) - \sin(\theta-\theta)]$

$= 5[\sin 2\theta] - \sin 0]$

$= 5\sin 2\theta - 0$

$= 5\sin 2\theta$

$\Rightarrow K = 5$

Q4 (i) $\cancel{B}$ $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

Sub in $(-B)$ for B.

$$\cos(A+(-B)) = \cos A \cos(-B) - \sin A \sin(-B)$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

(ii) Prove $\sin(A+B) = \sin A \cos B + \cos A \sin B$

Into $\cos(A-B) = \cos A \cos B + \sin A \sin B$
Sub ~~A~~ in $(90-A)$ for A.

$$\cos((90-A)-B) = \cos(90-A)\cos B + \sin(90-A)\sin B$$
$$\cos[90-(A+B)] = \sin A \cos B + \cos A \sin B$$
$$\Rightarrow \sin(A+B) = \sin A \cos B + \cos A \sin B$$

QED

Q5 (i) ~~Cos~~ Show $\sqrt{2\sin^2\theta + 6\cos^2\theta - 2} = 2\cos\theta$

$$\sqrt{2(1-\cos^2\theta) + 6\cos^2\theta - 2}$$

$$\sqrt{\cancel{2} - 2\cos^2\theta + 6\cos^2\theta - \cancel{2}}$$

$$\sqrt{4\cos^2\theta} = 2\cos\theta \quad \text{RHS}$$

Q5 (ii)

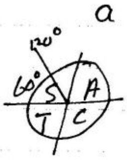
$$a \sin^2 2x + \cos 2x - b = 0$$

at $x = 0$

$$a \sin^2 2(0) + \cos 2(0) - b = 0$$
$$a \sin^2 0 + \cos 0 - b = 0$$
$$a(0)^2 + (1) - b = 0$$
$$\boxed{1 = b}$$

at $x = 60$

$$a \sin^2 2(60) + \cos 2(60) - b = 0$$
$$a \sin^2 120 + \cos 120 - b = 0$$

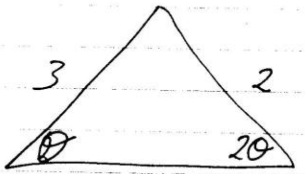


$$a \sin^2 60 + -\cos 60 - b = 0$$
$$a \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{2} - b = 0$$
$$a \left(\frac{3}{4}\right) - \frac{1}{2} - b = 0$$

but $\underline{\underline{b = 1}}$

$$a \left(\frac{3}{4}\right) - \frac{1}{2} - 1 = 0$$
$$a \left(\frac{3}{4}\right) - \frac{3}{2} = 0$$
$$a \left(\frac{3}{4}\right) = \frac{3}{2}$$
$$a = \frac{\cancel{3}}{2} \times \frac{4}{\cancel{3}}$$
$$\boxed{a = 2}$$

Q6



Sine Rule

$$\frac{\sin\theta}{2} = \frac{\sin 2\theta}{3}$$

$$\sin\theta = \frac{2}{3}(\sin 2\theta)$$

$$\sin\theta = 2/3\,(2\sin\theta\cos\theta)$$

$$\frac{\sin\theta}{2\sin\theta\cos\theta} = 2/3$$

$$\frac{1}{2\cos\theta} = 2/3$$

$$\frac{1}{\cos\theta} = \frac{4}{3}$$

$$\frac{3}{4} = \cos\theta$$

$$\cos^{-1}(3/4) = \theta$$

$$41.4^\circ = \theta$$

$$|\angle ACB| = 180 - \theta - 2\theta$$
$$= 180 - 41.4 - 82.8$$
$$= 55.8^\circ$$

$|AB| > 2$ as angle ACB is largest angle 55.8

$\Rightarrow$ Side $|AB|$ is largest side.

hence greater than 2.

Q7 (i) $\sin 2\theta = 1$

$$\Rightarrow 2\theta = \sin^{-1} 1$$
$$2\theta = 90^\circ$$
$$\theta = 45^\circ$$

$\therefore$ (a) $\sin 45 = \frac{1}{\sqrt{2}}$ and (b) $\tan 45 = 1$

(ii) $$\frac{\sin 4\theta\,(1-\cos 2\theta)}{\cos 2\theta\,(1-\cos 4\theta)} = \tan\theta$$

$$\frac{\sin 4\theta\,(1-(1-2\sin^2\theta))}{\cos 2\theta\,(1-(1-2\sin^2 2\theta))}$$

$$\frac{\sin 4\theta\,(2\sin^2\theta)}{\cos 2\theta\,(2\sin^2 2\theta}$$

$$\frac{2\cancel{\sin 2\theta}\cos 2\theta\ \cancel{2}\sin^2\theta}{\cancel{\cos 2\theta}\ \cancel{2}\ \underset{\sin 2\theta}{\cancel{\sin^2 2\theta}}}$$

$$\frac{2\sin^2\theta}{\sin 2\theta}$$

$$\frac{\cancel{2}\ \cancel{\sin\theta}\sin\theta}{\cancel{2}\ \cancel{\sin\theta}\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta.$$

Q8 (i) Area $= \frac{1}{2} ab \sin C$

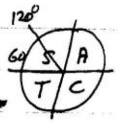
$$\tfrac{1}{2}(x)(x)\sin 4\theta = \tfrac{1}{2}(x)(x)\sin 2\theta.$$
$$\sin 4\theta = \sin 2\theta$$
$$2\sin 2\theta \cos 2\theta = \sin 2\theta$$
$$2\cos 2\theta = \frac{\sin 2\theta}{\sin 2\theta}$$
$$2\cos 2\theta = 1$$
$$\cos 2\theta = \tfrac{1}{2}$$
$$2\theta = \cos^{-1}(\tfrac{1}{2})$$
$$2\theta = 60^\circ$$
$$\theta = 30^\circ$$

(ii)

$$|AB|^2 + |DE|^2 = 24$$

$$|AB|^2 = x^2 + x^2 - 2xx\cos 4\theta$$
$$|DE|^2 = x^2 + x^2 - 2xx\cos 2\theta$$

$$x^2 + x^2 - 2x^2\cos 4(30) + x^2 + x^2 - 2x^2\cos 2(30) = 24.$$



$$4x^2 - 2x^2\cos 120 - 2x^2\cos 60 = 24$$
$$4x^2 - 2x^2(-\tfrac{1}{2}) - 2x^2(\tfrac{1}{2}) = 24$$
$$4x^2 + x^2 - x^2 = 24$$
$$4x^2 = 24$$
$$x^2 = 6$$
$$x = \sqrt{6}.$$

Q9 (i) $Area = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(2)^2(2\theta)$

$= 4\theta.$

(ii) $Area\ \Delta = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(2)(2) \sin 2\theta$

$= 2 \sin 2\theta.$

$$2 \sin 2\theta = \frac{3}{4}(4\theta)$$

$$\Rightarrow 2\sin 2\theta = 3\theta$$

$$2 \sin 2\theta = \sqrt{3}$$

$$\sin 2\theta = \sqrt{3}/2$$

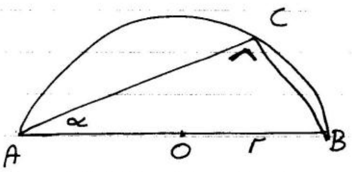
$$2\theta = \sin^{-1}(\sqrt{3}/2)$$

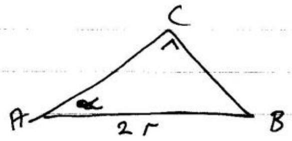
$$2\theta = \pi/3$$

$$\theta = \pi/6.$$

Q10

(i)

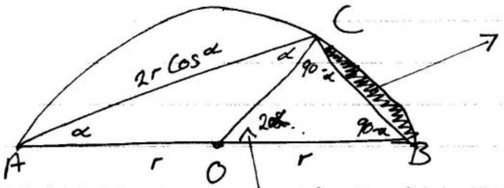




$\cos\alpha = \frac{AC}{2r}$

$2r\cos\alpha = AC$

(ii)



Shaded = Area Sector − Area Δ

$= \frac{1}{2} r^2 2\alpha - \frac{1}{2} r^2 \sin 2\alpha$.

$= \frac{1}{2} r^2 \left[2\alpha - \sin 2\alpha\right]$

$180 - (90-\alpha) - (90-\alpha) = 2\alpha$.

Area $\Delta ABC = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(2r)(2r\cos\alpha)\sin\alpha$.

$= 2r^2 \cos\alpha \sin\alpha$.

$= r^2 \; 2\cos\alpha \sin\alpha$

$= r^2 \; \sin(\alpha+\alpha) - \sin(\alpha-\alpha)$

$= r^2 \sin 2\alpha$.

$\boxed{\text{Area of Sector} = \frac{1}{2} r^2 \theta}$

Area Semi Circle $= \frac{1}{2} r^2 \pi = \frac{\pi}{2} r^2$.

$\frac{1}{2}\left(\frac{\pi}{2}\right)\cancel{r^2} = \cancel{r^2}\sin 2\alpha + \frac{1}{2}\cancel{r^2}(2\alpha - \sin 2\alpha)$ $\left[\frac{1}{2} \text{ of Semi} = \text{area } \Delta + \text{Shaded}\right]$

$\frac{\pi}{4} = \sin 2\alpha + \alpha - \frac{1}{2}\sin 2\alpha$. $(\times 2)$

$\frac{\pi}{2} = 2\sin 2\alpha + 2\alpha - \sin 2\alpha$

$\pi/2 = \sin 2\alpha + 2\alpha$

Q.E.D.