Test Questions

B Questions

Q1 $P(\text{win}) = 2/3$

To win two sets

$\Rightarrow$ win win $\rightarrow 2/3 \times 2/3 = 4/9$

or (+)

lose win win $\rightarrow 1/3 \times 2/3 \times 2/3 = 4/27$

or (+)

win lose win $\rightarrow 2/3 \times 1/3 \times 2/3 = 4/27$

$4/9 + 4/27 + 4/27 = 20/27$

Q2

$P(\text{All fit}) = 7/10$ $\quad P(\text{win}) = 9/10$

$P(\text{win}) = 4/10$

P(win next home Game) $\Rightarrow$

all fit and win $\rightarrow 7/10 \times 9/10 = 63/100$

or (+)

not all fit and win $\rightarrow 3/10 \times 4/10 = 12/100$

$63/100 + 12/100 = 75/100 = 3/4$

Q4 (i) $P(\text{Not Biol}) = 21/56 = 3/8$

(ii) At least 2 = $7+6+9+4 = 26$

$P(\text{Not Biol}) = 4/26 = 2/13$

(iii) Pick 2 students $= \binom{56}{2}$

The 2 Study Physics $= \binom{28}{2}$

$$P(\text{Both Study Physics}) = \frac{\binom{28}{2}}{\binom{56}{2}} = \frac{27}{110}$$

(iv) 2 picked from Chem = 25 students.
1 of the 2 does Biol. $(13/25)$

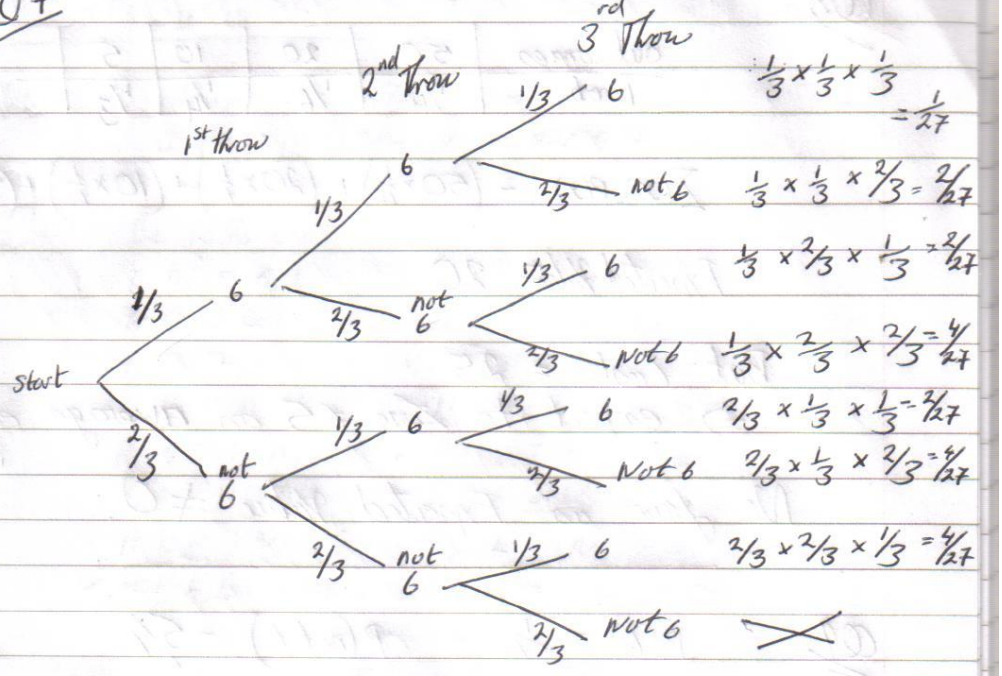
Biol + Not Biol $\rightarrow \frac{13}{25} \times \frac{12}{24} = 13/50$

or (+)

Not Biol + Biol. $\rightarrow \frac{12}{25} \times \frac{13}{24} = 13/50$

$$13/50 + 13/50 = 26/50 = 13/25.$$

Q7



At least 16 ⇒ 1 or 2 or 3 sixes.

$= 1/27 + 2/27 + 2/27 + 4/27 + 2/27 + 4/27 + 4/27 = 19/27$.

(ii) $P(A) = 2/3$ $P(A \cup B) = 3/4$ $P(A \cap B) = 5/12$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$3/4 = 2/3 + x - 5/12$

$3/4 - 2/3 + 5/12 = x$

$1/2 = x$

$P(B) = 1/2$.

Q8

outcomes	50	20	10	5
Prob	1/4	1/6	1/4	1/3

$$\sum(x).P(x) = \left(50 \times \tfrac{1}{4}\right) + \left(20 \times \tfrac{1}{6}\right) + \left(10 \times \tfrac{1}{4}\right) + \left(5 \times \tfrac{1}{3}\right)$$

Expected Value = 20

But Cost = 25

⇒ expect to Lose €5 on average each game.

No fair as Expected Value ≠ 0.

Q9 $P(6) = 1/6$ $P(\text{not } 6) = 5/6$

(i) $P(\text{2 six in 6 rolls}) = \binom{6}{2}\left(\frac{1}{2}\right)^2\left(\frac{5}{6}\right)^4$

$= 1.808$

$= 0.2$

(ii) The 2nd six on 7th roll.

⇒ 1 six in first 6 rolls.

$= \binom{6}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^5 = 0.40188$

$P(6) = 1/6$

$\therefore \quad 0.40188 \times \frac{1}{6} = 0.06698$

$= 0.67.$

C Questions

Q1 (i) other 2 paths are
A B E H and A C E H

(ii) The Paths are:

A B D G L	A C F K Q
A B D G M	✓A C F K P
A B D H M	✓A C F J P
A B D H N	A C F J N
A B E H M	✓A C E J P
A B E H N	A C E J N
A B E J N	A C E H N
✓ A B E J P	A C E H M

P(Passes through H or J) = 12/16 = 3/4

(iii) P(Land on N) = 6/16 = 3/8

(iv) P(Land on P) = 4/16 = 1/4
and 2nd one land on P = 1/4

Both land on P = 1/4 x 1/4 = 1/16.

Q2 $P(\text{Success}) = 0.7 = P(\text{fail}) = 0.3$

(i) $P(\text{Scores 1st on 3rd attempt})$

$\Rightarrow$ Miss and Miss and Score

$= 0.3 \times 0.3 \times 0.7 = 0.063$

(ii) $P(\text{Scores 3 in 5 attempts}) = \binom{5}{3}\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2$

$= 0.3087$

$= 0.309$

(iii) $P(3^{rd} \text{ goal on } 7^{th} \text{ attempt})$

$\Rightarrow$ 2 goals in 6.

$= \binom{6}{2}(0.7)^2(0.3)^4$

$= 0.059535$

Goal on $7^{th} = 0.059535 \times 0.7$

$= 0.0416745$

$= 0.042$

Q3 (a) $P(A) = \left(\frac{13}{25}\right)$, $P(B) = \frac{9}{25}$ $P(A|B) = \frac{5}{9}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

(i) $P(A \text{ and } B) = P(A \cap B)$

$$\frac{5}{9} = \frac{P(A \cap B)}{9/25}$$

$$\frac{5}{9} \times \frac{9}{25} = P(A \cap B)$$

$$\frac{1}{5} = P(A \cap B)$$

(ii) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$$P(B|A) = \frac{1/5}{13/25} = \frac{1}{5} \times \frac{25}{13} = \frac{5}{13}$$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{13}{25} + \frac{9}{25} - \frac{1}{5} = \frac{17}{25}$$

Q3(b) $P(6) = P$

$\Rightarrow P(\text{Not } 6) = \frac{1-P}{5}$

Fair dice $P(\text{any No}) = 1/6$

To get score of 7: (3,4) (4,3) (5,2) (2,5) (6,1) (1,6)

all independent of P.

$\frac{6}{36} = \frac{1}{6}$.

Q4

$\mu = 48 \qquad \sigma = 8$

(i) $P(x > 60) \qquad z = \frac{60-48}{8} = 1.5$

$\therefore P(z > 1.5) = 1 - 0.9332$

$= 0.0668$

(ii) $P(x < 35) \qquad z = \frac{35-48}{8} = -1.625$.

$\therefore P(z < -1.625) = 1 - 0.9484$

(-1.63)

$= 0.0516$

Q5 4 Red 6 Green Total = 10

Draw 4.

(i) $P(\text{all Green}) = \frac{\binom{6}{4}}{\binom{10}{4}} = \frac{1}{14}$

(ii) P(at least 1 of each colour)

$\Rightarrow$ 1R and 3G or 2R and 2G or 3R and 1G

$$= \frac{\binom{4}{1}\cdot\binom{6}{3}}{\binom{10}{4}} + \frac{\binom{4}{2}\binom{6}{2}}{\binom{10}{4}} + \frac{\binom{4}{3}\binom{6}{1}}{\binom{10}{4}}$$

$$= \frac{80}{210} + \frac{90}{210} + \frac{24}{210} = \frac{97}{105}.$$

(iii) P(at least 2 Green)

$\Rightarrow$ 2G and 2R or 3G and 1R or 4G

$$\frac{\binom{6}{2}\cdot\binom{4}{2}}{\binom{10}{4}} + \frac{\binom{6}{3}\binom{4}{1}}{\binom{10}{4}} + \frac{\binom{6}{4}}{\binom{10}{4}}$$

$$\frac{90}{210} + \frac{80}{210} + \frac{15}{210} = \frac{37}{42}.$$

(iv) P(at least 2G and at least 1 of each colour)

$\Rightarrow$ 2G, 2R or 3G, 1R

$$\frac{\binom{6}{2}\cdot\binom{4}{2}}{\binom{10}{4}} + \frac{\binom{6}{3}\binom{4}{1}}{\binom{10}{4}}$$

$$\frac{80}{210} + \frac{90}{210} = 17/21$$

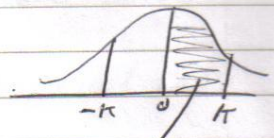
Independent $\Rightarrow$ $P(A \cap B) = P(A) \times P(B)$

$$\frac{17}{21} = \frac{37}{42} \times \frac{97}{105}$$

$$\frac{17}{21} \neq \frac{3589}{4410} \Rightarrow \text{Not Independent}$$

Q6

(i) $P(-H \leq Z \leq H) = 0.8438$



$0.8438 \div 2 = 0.4219$.

$0.4219 + 0.5 = 0.9419$

$\Rightarrow$ (from tables) $Z = 1.42$.

(ii) $P(X) = 2/3$ $P(X|Y) = 2/3$ $P(Y) = 1/4$

(a) $P(X \cap Y)$: $P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$

$$2/3 = \frac{P(X \cap Y)}{1/4}$$

$$2/3 \times 1/4 = P(X \cap Y)$$

$$1/6 = P(X \cap Y)$$

(b) $P(Y|X) = \frac{P(Y \cap X)}{P(X)}$

$$P(Y|X) = \frac{1/6}{2/3}$$

$$P(Y|X) = 1/6 \times 3/2 = 1/4$$

Q7 (a) (i) all sum to 1

$$0.1 + a + b + 0.2 + 0.1 = 1$$
$$a + b = 0.6.$$

(ii) $E(x) = 2.9$.

$$E(x) = \Sigma x \cdot P(x) = 0.1 + 2a + 3b + 0.8 + 0.5 = 2.9$$
$$2a + 3b + 1.4 = 2.9$$
$$2a + 3b = 1.5.$$

Solve Sim Eqn

$$a + b = 0.6 \quad (\times -2)$$
$$2a + 3b = 1.5$$
$$-2a - 2b = -1.2$$
$$2a + 3b = 1.5$$
$$b = 0.3.$$

$$a + b = 0.6$$
$$a + 0.3 = 0.6$$
$$a = 0.3$$

(b) 16 Girls 8 boys

P(Boy studies F) $= \frac{x}{8}$ P(Girl Studies F) $=$

$x + y = 12$

$\frac{y}{16} = \frac{3}{2}\left(\frac{x}{8}\right)$

$\frac{y}{16} = \frac{3x}{16}$

$y = 3x$

$x + 3x = 12$

$4x = 12$

$x = 3$

$\Rightarrow$ 3 boys $\therefore$ 9 Girls.

Q10

(i) mean, mode, median are all the same.

(ii) Bell shaped, Symmetrical, empirical rule.

(iii) $\mu = 12000 \qquad \sigma = 300$

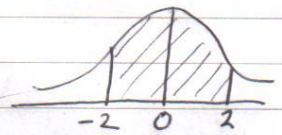
(a) $P(x < 11400) \qquad z = \frac{11400 - 12000}{300} = -2$

$P(z < -2) = 1 - 0.9772$

$= 0.0228.$

(b) $P(11400 < x < 12600) \qquad z = -2$

$z = \frac{12600 - 12000}{300} = 2.$



$P(-2 \leq z < 2) = 0.9772 - 0.0228$

$= 0.9544.$

(c) Longer than 12600

$P(x > 12600) \qquad z = 2$

$P(z > 2) = 1 - 0.9772 = 0.0228.$

$5000 \times 0.0228 = 114$ bulbs